NAVOIY DAVLAT KONCHILIK VA TEXNOLOGIYALAR UNIVERSITETI

"OLIY MATEMATIKA VA AT" KAFEDRASI

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Mavzu:Differensial tenglamalar va tenglamalar sistemasini operatsion hisob yordamida yechish .

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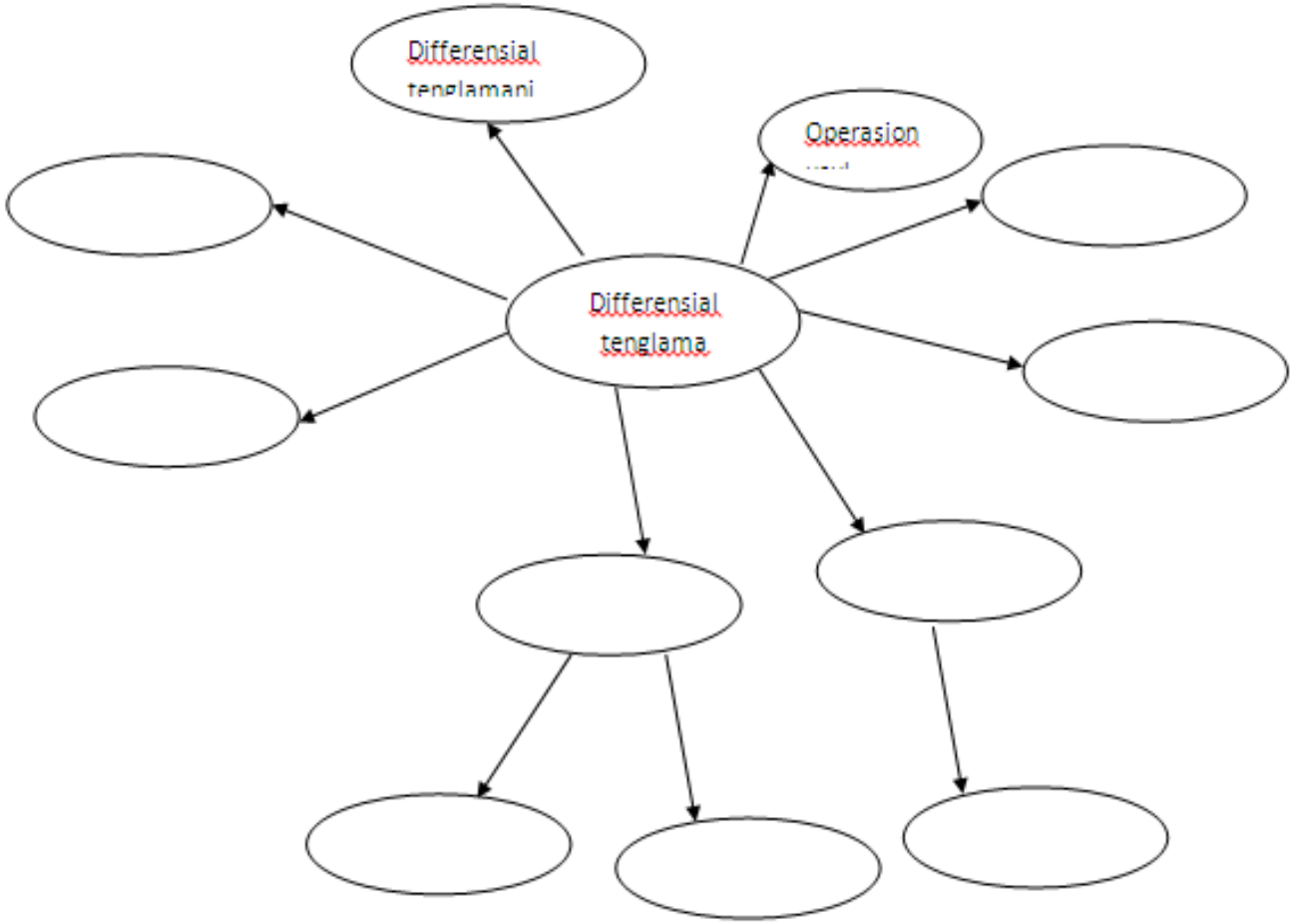
Reja

Originalni tasvir bo‘yicha tiklash usullari.

Differensial tenglamalarni operasion usul bilan yechish

Differensial tenglamalarni sistemasini operasion usul bilan yechish


Differensial
Operasion
Differensial
tenglama

Parametr bo'yicha differensiallash teoremasi.

Agar $f(t,x) \leftarrow F(p,x)$ bo'lsa, u holda

$$\frac{\partial f(t,x)}{\partial x} \leftarrow \frac{\partial F(p,x)}{\partial x}$$

Isboti. Haqiqatan ham,

$$F(p,x) = \int_0^\infty e^{-pt} f(t,x)dt$$

bo'lganligi uchun integralni x parametr bo'yicha differensiallab,

$$\frac{\partial F(p,x)}{\partial x} = \int_0^\infty e^{-pt} \frac{\partial f(t,x)}{\partial x} dt$$

ni hosil qilamiz, bundan,

$\frac{\partial f(t,x)}{\partial x} \leftarrow \frac{\partial F(p,x)}{\partial x}$

Bu xossa ko'p sondagi tasvirlarni hosil qilish imkonini beradi.

Misol. $f(t) = t^n e^{\alpha t}$ orginalning tasvirini toping.

Yechish. $e^{\alpha t} \leftarrow \frac{1}{p-\alpha}$ moslikning ikkala tomonini α parametr bo'yicha differensiallab, quyidagilarni hosil qilamiz:

$$te^{\alpha t} \leftarrow \frac{1}{(p-\alpha)^2}; \quad t^2 e^{\alpha t} \leftarrow \frac{2}{(p-\alpha)^3};$$

$$t^3 e^{\alpha t} \leftarrow \frac{3!}{(p-\alpha)^4}; \qquad t^n e^{\alpha t} \leftarrow \frac{n!}{(p-\alpha)^{n+1}};$$

Orginalni differensiallash teoremasi

Agar $f(t) \leftarrow F(p)$ bo'lib, $f'(t)$ orginal bo'lsa, u holda

$$f'(t) \leftarrow pF(p) - f(0)$$

bo'ladi.

Isboti: $f'(t)$ hosila uchun Laplas almashtirishni yozamiz

$$f'(t) = \int_0^{\infty} f'(t)e^{-pt}dt$$

Bo'laklab integrallab va $u = e^{-pt}$, $du = -pe^{-pt}dt, dv = f'(t)dt, v = f(t)$ deb olib,

$f'(t) \leftarrow f(t)e^{-pt}|_0^{\infty} + p\int_0^{\infty} f(t)e^{-pt}dt$ (1)

ni hosil qilamiz.

$f(t)$ uchun $f(t) \leq Me^{S_0 t}$ ga egamiz, shu sababli, agar Rep>s_0 bo'lsa, u holda $t \to \infty$ da

$$|f(t)e^{-pt}| < Me^{(S_0 - Rep)t} \to 0$$

Natijada (1) moslik birinchi qo'shiluvchida –f(0) qoladi va (1) munosabat uzil – kesil ushbu ko'rinishni oladi:

$$f'(t) \leftarrow pF(p) - f(0) \qquad (2)$$

Xususan, agar $f(0) = 0$ bo'lsa, u holda

$$f'(t) \leftarrow pF(p)$$

Teoremani takror tatbiq etib, quyidagilarni hosil qilamiz:

$$f"(t) \leftarrow p[pF(p) - f(0)] - f'(0)$$
$$= p^2F(p) - pf(0) - f'(0)$$
$$f'''(t) \leftarrow p[p^2F(p) - pf(0) - f'(0)] - f"(0)$$
$$= p^3F(p) - p^2f(0) - pf'(0) - f"(0)$$

(2) formulani n-1 marta tatbiq etib quyidagini umumiy formulani hosil qilamiz:

$$f^n(t) \leftarrow p^n F(p) - p^{n-1} f(0) - p^{n-2} f'(0) - p^{n-3} f''(0) - \ldots - f^{n-1}(0) \quad (3)$$

Xususiy holda funksiyaning va uning hosilalarining barcha boshlang'ich qiymatlari nolga teng bo'lganda $f^n(t) \leftarrow p^n F(p)$ ni hosil qilamiz. Shunday qilib, boshlang'ich qiymatlar nol bo'lganda orginalni n- karra differensiallash uning tasvirini p^n ga ko'paytirishga keltiriladi.

Differensial tenglamalar va tenglamalar sistemasini operasion usul yordamida yechish

O`zgarmas $a_0, a_1, \ldots, a_{n-1}, a_n$ koefisiyentli n-tartibli chiziqli differensial tenglama berilgan bo`lsin.

$$a_0 \frac{d^n x}{dt^n} + a_1 \frac{d^{n-1} x}{dt^{n-1}} + \ldots + a_{n-1} \frac{dx}{dt} + a_n x(t) = f(t) \qquad (4)$$

Bu tenglamaning

$$x(0) = x_0, x'(0) = x'_0, \ldots, x_0, x^{(n-1)}(0) = x^{(n-1)}_0 \qquad (5)$$

Boshlang`ich shartlarni qanoatlantiruvchi $t \geq 0$ uchun $x = x(t)$ yechimini topish talab qiinadi.

Qo`yilgan masalani ilgari bunday yechgan edik: (4) tenglamaning n ta ixtiyoriy o`zgarmassiga ega bo`lgan umumiy yechimini topib, so`ngra o`zgarmaslarni (5) boshlang'ich sahrtlarni qanoatlantiradigan qilib aniqlangan edik. Bu yerda masalani yechishning soddaroq metodini – operasion hisob metodini bayon qilamiz.(4) tenglamaning (5) shartlarni qanoatlantiradigan x(t) yechimning L – tasvirini topamiz. Bu L- tasvirni $\bar{x}(p)$ bilan belgilaymiz. Shunday qilib $\bar{x}(p) \to x(t)$

(4) tenglama yechimlarining tasvirlari va uning n-tartibgacha hosilalari mavjud deb faraz qilamiz.(4) tenglikning ham ma hadlarini e^{-pt}ga ko`paytiramiz (bu yerda p=a+ib) va t bo`yicha 0 dan ∞ gacha chegarada integrallaymiz:

$$a_0 \int_0^{\infty} e^{-pt} \frac{d^n x}{dt^n} \quad \text{dt+} \quad a_1 \int_0^{\infty} e^{-pt} \frac{d^{n-1} x}{dt^{n-1}} \quad \text{dt+…+}$$

$$a_n \int_0^{\infty} e^{-pt} x(t) \quad \text{dt=} \quad \int_0^{\infty} e^{-pt} f(t) \quad \text{dt} \qquad (6)$$

Bu tenglikning chap tomonida x(t) funksiyaning va hosilalarining L- tasvirlari, o`ng tomonida esa f(t) funksiayning L- tasviri turibdi. Uni F(p) bilan belgilaymiz. Demak, (6) tenglikni bunday qayta yozish mumkin:

$$a_0 L\left\{\frac{d^n x}{dt^n}\right\} + a_1 L\left\{\frac{d^{n-1} x}{dt^{n-1}}\right\} + \ldots + a_n L\{x(t)\} = L\{f(t)\}$$

Bu tenglikdagi funksiyaning tasvirlari va hosilalarining tasvirlari o`rniga

$$pF(p) - f(o) \to f'(t)$$

$$p^2 F(p) - pf(0) - f'(0) \to f''(t)$$

$$p^n F(p)$$

$$- \left[p^{n-1} f(0) + p^{n-2} f'(0) + \cdots + pf^{(n-2)}(0) + f^{(n-1)}(0)\right]$$

$$\to f^n(t)$$

ifodalarni qo`yib, ushbuni hosil qilamiz:

$$a_0\left\{p^n \bar{x}(p) - \left[p^{n-1} x_0 + p^{n-2} x'_0 + p^{n-3} x'_0 + \ldots + x_0^{(n-1)}\right]\right\} +$$

$$+ a_1\left\{p^{n-1} \bar{x}(p) - \left[p^{n-2} x_0 + p^{n-3} x'_0 + \ldots + x_0^{(n-2)}\right]\right\} + \qquad (7)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_{n-1}(p\bar{x}(p) - x_0) + a_n \bar{x}(p) = F(p)$$

(7) Tenglama yordamchi tenglama yoki tasvirlovchi tenglama deb ataladi. Bu tenglamada noma`lim $\bar{x}(p)$ tasvir bo`lib, u shu tenglamadan topiladi. (7) tenglamani $\bar{x}(p)$ ishtirok etgan hadlari chap qismida qoldirib, quyidagicha almashtirib yozamiz:

$$
\begin{aligned}
&\bar{x}(p)(a_0 p^n + a_1 p^{n-1} + \ldots + a_{n-1} p + a_n) = \\
&= a_0 (p^{n-1} x_0 + p^{n-2} x_0' + \ldots + x_0^{(n-1)}) + \\
&+ a_1 (p^{n-2} x_0 + p^{n-3} x_0' + \ldots + x_0^{(n-2)}) + \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&+ a_{n-2} (p x_0 + x_0') + a_{n-1} x_0 + F(p)
\end{aligned}
\qquad (7')
$$

(7) Tenglikning chap qismidagi $\bar{x}(p)$ ning koeffisiyenti p ga nisbatan n- darajali ko`phad buni hosil qilish uchun (4) tenglamaning chap qismida hosilalar o`rniga p ning mos darajalarini qo`yish kifoya. Uni $\varphi_n(p)$ bilan belgilaymiz:

$$\varphi_n(p) = a_0 p^n + a_1 p^{n-1} + \cdots + a_{n-1} p + a_n \qquad (8)$$

(7') tenglamaning o`ng qismi quyidagicha tuziladi:
a_{n-1} koeffisiyent x_0 ga ko`paytiriladi,
a_{n-2} koeffisiyent $px_0 + x'_0$ ga ko`paytiriladi,
……
$a_1$koeffisiyent $p^{n-1}x_0 + p^{n-3}x'_0 + \cdots + x_0^{(n-1)}$ ga ko`paytiriladi, $a_0$ koeffisiyent $p^{n-1}x_0 + p^{n-2}x'_0 + \cdots + x_0^{(n-1)}$ ga ko`paytiriladi.

Bu ko`paytmalarning hammasi qo`shiladi. Bunga differensial tenglama o`ng qismining tasviri F(p) ham qo`shiladi. (7') tenglikning o`ng qismidagi F(p) dan boshqa hamma hadlar o`xshashlari yig'ishtirilganda so`ng p ga nisbatan n-1 darajali ko`phadni tashkil etadi.Uni $\tau_{n-1}(p)$ bilan belgilaymiz. Shunday qilib (4') tenglamani bunday yozish mumkin:

$$\bar{x}(p)\varphi_n(p) = \tau_{n-1}(p) + F(p)$$

Bu tenglamadan $\bar{x}(p)$ ni aniqlaymiz:

$$\bar{x}(p) = \frac{\tau_{n-1}(p)}{\varphi_n(p)} + \frac{F(p)}{\varphi_n(p)} \qquad (9)$$

Shunday qilib $\bar{x}(p)$ (4) tenglamaning (5) boshlang'ich shartlarini qanoatlantiruvchi x(t) yechimining tasviridir.

Agar biz (4) tenglamaning nolga teng boshlang'ich shartlar:

$x_0 = x'_0 = x''_0 = \cdots = x_0{}^{n-1} = 0$ bo`yicha yechimini topsak, bu holda, (9) tenglikda $\tau_{n-1}(p) = 0$ bo`ladi va u tenglik ushbu ko`rinishga keladi:

$$\bar{x}(p) = \frac{F(p)}{\varphi_n(p)}$$

Yoki

$$\bar{x}(p) = \frac{F(p)}{a_0 p^n + a_1 p^{n-1} + \cdots + a_n} \quad (9')$$

Misol. Agar $x(0) = 0, x'(0) = 0$

$x" - 2x' - 3x = e^{3t}$ bo`lsa tenglamaning yechimini toping.

Yechish. Berilgan tenglamadan operatorli tenglamaga o`tamiz: $x(t) \leftarrow X(p)$

U holda $x'(t) \leftarrow pX(p), x"(t) \leftarrow p^2X(p)$

Tasvirlar jadvalidan $e^{3t} \leftarrow \frac{1}{p-3}$ ni topamiz.

Operatorli tenglama quyidagi ko`rinishda bo`ladi:

$$p^2X(p) - 2pX(p) - 3X(p) = \frac{1}{p-3}$$

Yoki

$$(p^2 - 2p - 3)X(p) = \frac{1}{p-3}$$

bundan:

$$X(p) = \frac{1}{(p-3)^2(p+1)}$$

Ratsional kasrni eng sodda kasrlarga yoyamiz:

$$\frac{1}{(p-3)^2(p+1)} = \frac{A}{(p-3)^2} + \frac{B}{p-3} + \frac{C}{p+1}$$

$$1 = A(p+1) + B(p-3)(p+1) + C(p-3)^2$$

A,B,C koeffisiyentlarni quyidagi tenglamalar sistemasidan topamiz:

$$\begin{array}{l|l} p=-1 & 1=16C \\ p=3 & 1=4A \\ p^2=0 & 0=B+C \end{array}$$

Demak, $A=\frac{1}{4}, C=\frac{1}{16}, B=-\frac{1}{16}$

Shunday qilib,

$$X(p)=\frac{1}{4(p-3)^2}-\frac{1}{16(p-3)}+\frac{1}{16(p+1)}$$

Tasvirlar jadvalidan foydalanib xususiy yechimini (orginalini) topamiz:

$$x(t)=\frac{1}{4}te^{3t}-\frac{1}{16}e^{3t}+\frac{1}{16}e^{-t}$$

Differensiallovchi zveno. Chiqish kattaligi kirish parametrlarining o'zgarish tezligiga proporsional bo'lgan zveno differensiallovchi zveno deyiladi. Bu ideal differensiallovchi zvenoning xususiyatlari

$$y(t) = K \cdot \frac{dx}{dt} \qquad (10)$$

tenglama bilan ifodalanadi. Bundan K- uzatish koeffisiyenti. Unga elektr sig'im, induktivlik, taxogenerator (agar kirish kattaligi o'qning aylanish tezligi emas, burchak burulishi bo'lsa) misol bo'la oladi. (10) tenglamani Laplas bo'yicha o'zgartirib, zvenoning uzatish funksiyasini aniqlaymiz

$$W(p) = \frac{y(p)}{x(p)} = Kp$$

Tebranuvchi zveno. Ushbu zvenoning chiqish va kirish kattaliklari o'rtasidagi bog'lanish ikkinchi tartibli differensial tenglama orqali ifodalanadi:

$$T^2\frac{d^2y}{dt^2}+2\xi T\frac{dy}{dt}+y(t)=K\cdot x(t) \qquad (11)$$

Bu yerda $0<\xi<1$ oralig'idagi qiymatga ega bo'lib, so'nish darajasi (koeffitsiyenti) deyiladi. Elektr tebranuvchi zanjir, elastik mexanik tizim bu zvenoga misol bo'la oladi. (11) tenglamani Laplas almashtirishi orqali o'zgartirib

$$(T^2p^2+2\xi Tp+1)y(p)=Kx(p)$$

zvenoning uzatish funksiyasi aniqlanadi

$$W(p)=\frac{y(p)}{x(p)}=\frac{K}{T^2p^2+2\xi Tp+1}$$

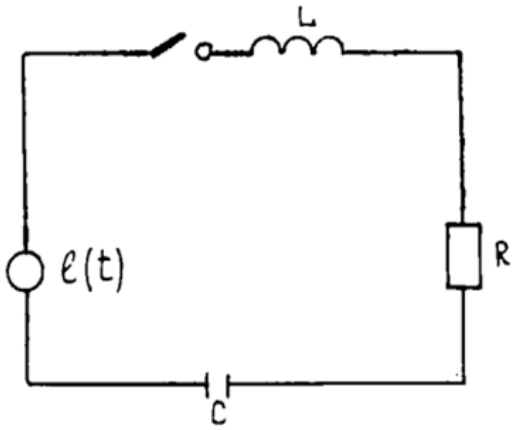
2.Masala. Elektr zanjiridagi tebranishlar haqidagi masala.

Elektr yurituvchi kuchi e(t) ga teng bo`lgan manbaga ketma-ket ulangan L induktivlik g`altagi, R_{om} qarshilik va C sig'imdan iborat konturulangan. Agar boshlang'ich paytda konturdagi tok va kondesator zaryadi nolga teng bo`lsa, zanjirdagi I tok t vaqtning funksiyasi sifatida toping. (1-shakl)

Yechish: Kirxgof qonuniga binoan zanjirdagi elektr yurituvchi kuch induktivlikdagi, qarshilikdagi va sig`imdagi kuchlanishlar pasayishi yig`indisiga teng:

$$\mathcal{E}(t) = u_L + u_R + u_C$$

Ular I tok bilan quyidagi munosabatlar orqali bog`langan:



1-shakl

$$u_L = L\frac{di}{dt},\ u_R = Ri,\ u_C = \frac{1}{C}\int_0^t i(\tau)d\tau$$

Eslatma. Bu yerda oxirgi tenglik tok va kondesator zaryadi orasidagi munosabatdan topiladi:

$i = \frac{dq}{dt}$ bundan $q = \int_0^t i(\tau)d\tau + q_0$ so`ngra $u_C = \frac{q}{C}$ bo`lgani uchun

$U_c \frac{1}{C}\int_0^t i(\tau)d\tau + \frac{q_0}{C}$ berilgan masalada shartga ko`ra $q_0 = 0$

Shunday qilib, quyidagi tenglama hosil bo`ladi:

$$e(t) = L\frac{di}{dt} + Ri + \frac{1}{C}\int_0^t i(\tau)d\tau$$

Bu integro – differensial tenglamadir, u tenglamalarning eng murakkab turlaridan biriga mansubdir. Lekin mazkur holda uni t bo`yicha differensiallab, o`zgarmas koeffisiyentli ikkinchi tartibli oddiy differensial tenglama o`tish mumkin:

$$L\frac{d^2i}{dt^2} + \mathrm{R}\frac{di}{dt} + \frac{1}{C}i = 0$$

Ikki holni ko`ramiz.

1. $e(t) = E = const$ bu holda $\frac{de}{dt} = 0$ va oxirgi tenglama bir jinsli

$$\mathrm{L}\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$

tenglamaga aylanadi. Bu differensial tenglama tok uchun mexanik tebranishlarning muhitning qarshiligini hisobga olingandagi tenglamasiga o`xshaydi. Uni

$i(0)=0, i'(0)=\frac{E}{L}$ Boshlang`ich shartlarda operasion usul yordamida yechaylik.

Operator tenglama

$$\left[p^2 I(p)-\frac{E}{L}\right]+\frac{R}{L}pI(p)+\frac{1}{LC}I(p)=0$$

Operator yechim esa

$$I(p)=\frac{E}{L}\frac{1}{p^2+\frac{R}{L}p+\frac{1}{LC}}$$

ko`rinishda bo`ladi.

$\frac{R}{L}=2\delta$ deb belgilab, quyidagi uch holni kor`aylik:

1-hol. $\frac{1}{LC}-\frac{R^2}{4L^2}=w_1^2>0$ Buholda tasvirdan orginalga o`tib $i(t)$

tok uchun so`nuvchi elektr tebranishlarni ifodalovchi quyidagi

$$i(t)=\frac{E}{w_1 L}e^{-\delta t}\sin w_1 t$$ Yechimini hosil qilamiz

2- hol. $\frac{R^2}{4L^2} - \frac{1}{LC} = \beta^2 > 0$ Bu holda $i(t) = \frac{E}{\beta L} e^{-\delta t} \sin \beta t$

I tok davriy va zanjirda hech qanday tebranishlar sodir bo`lmaydi.

3-hol. $\frac{R^2}{4L^2} - \frac{1}{LC} = 0$ Bu holda operator yechim

$$I(p) = \frac{E}{L} \frac{1}{(p+\delta)^2}$$

Demak,

$$I(p) = \frac{E}{L} te^{-\delta t}$$

ya`ni bu holda ham $i(t)$ tok davriy bo`lmay, elektr tebranishlar bo`lmaydi.

II. $e(t) = E \sin \omega t.$ Bu holda $\frac{de}{dt} = E\omega \cos \omega t$

va quyidagi chiziqi ikkinchi tartibli bir jinsli bo`lmagan tenglama hosil bo`ladi:

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = \frac{E}{L} \omega \cos \omega t$$

Boshlang`ich shartlar quyidagicha:

$$i(0) = 0; \mathrm{i}'(0) = 0$$

bo`lsin, $\frac{1}{L}-\frac{R^2}{4L}=\omega_1^2>0$ bo`lgan holni ko`raylik. Operator tenglama

$$p^2I(p)+\frac{R}{L}pI(p)+\frac{1}{LC}I(p)=\frac{E\omega}{L}\frac{p}{p^2+\omega^2}$$

Operator yechim esa

$$I(p)=\frac{E\omega}{l}\frac{p}{(p^2+\frac{R}{L}p+\frac{1}{LC})(p^2+\omega^2)}$$

Quyidagi

$$\frac{R}{2L}=\delta,\ \frac{1}{\mathrm{LC}}=\omega_0^2,\omega_1^2=\omega_0^2-\delta^2$$

belgilashlarni kiritib, tasvirdan orginalga o`tsak:

$$i(t)=\frac{E\omega}{L}\cdot\frac{1}{(\omega^2-\omega_0^2)+4\delta^2\omega^2}\left\{e^{-\delta t}\left[(\omega^2-\omega_0^2)\cos\omega t-\right.\right.$$

$$\left.\left.-\frac{\delta}{\omega_1}(\omega^2+\omega_0^2)\sin\omega_1 t\right]-(\omega^2-\omega_0^2)\cos\omega t+2\delta\omega\sin\omega t\right\}.$$

Yana quyidagi

$$\omega^2 - \omega_0^2 = \omega^2 - \frac{1}{LC} = \frac{\omega}{L}\left(\omega L - \frac{1}{\omega C}\right) = \frac{\omega}{L} G,\ C = \omega L - \frac{1}{\omega C},$$

$$(\omega^2 - \omega_0^2)^2 + 4\delta^2\omega^2 = \frac{\omega^2}{L^2} G^2 + \frac{R^2}{L^2}\ \omega^2 = \frac{\omega^2}{L^2}(G^2 + R^2) = \frac{\omega^2}{L^2} \cdot Z^2,$$

$$Z^2 = G^2 + R^2,$$

$$\omega^2 + \omega_0^2 = \omega^2 + \frac{1}{LC} = \frac{\omega}{L}\left(\omega L + \frac{1}{\omega C}\right) = \frac{\omega}{L} G_0,\ G_0 = \omega L + \frac{1}{\omega C}$$

belgilashlarni kiritsak, u holda

$$i(t) = \frac{EL}{\omega Z^2}\left\{e^{-\delta t}\left[\frac{\omega}{L} G \cos\omega_1 t - \frac{\delta}{\omega_1}\frac{\omega}{L} G_0 \sin\omega_1 t\right] - \right.$$

$$\left. - \frac{\omega}{L} G \cos\omega t + 2\delta\omega \sin\omega t\right\} =$$

$$= \frac{E}{Z^2}\left\{\frac{e^{-\delta t}}{\omega_1}\left[\omega_1 G \cos\omega_1 t - \delta G_0 \sin\omega_1 t\right] - G\cos\omega t - R \sin\omega_1 t\right\}$$

Endi

$$\frac{R}{Z} = \cos\gamma, \quad \frac{G}{Z} = \sin\gamma$$

$$\frac{\omega_1 G}{\sqrt{\omega_1^2 G^2 + \delta^2 G_0^2}} = \sin\gamma_1, \quad \frac{\delta G_0}{\sqrt{\omega_1^2 G^2 + \delta^2 G_0^2}} = \cos\gamma_1,$$

Demak, u holda

$$i(t) = \frac{-E\sqrt{\omega_1^2 G^2 + \delta G_0^2}}{\omega_1 z^2} e^{-\delta t} \sin(\omega_1 t - \gamma_1) + \frac{E}{Z} \sin(\omega_1 t - \gamma).$$

Quyidagi

$$\frac{\sqrt{\omega_1^2 G^2 + \delta^2 G^2}^2}{Z} = \omega_0$$

tenglik o`rinli ekanini ko`rsatamiz. Xaqiqatan ham, ildiz ostidagi ifodani quyidagicha o`zgartirsak,

$$\omega_1^2 G^2 + \delta^2 G_0^2 = (\omega_0^2 - \delta^2) G^2 + \delta G_0^2 = \omega_0^2 G^2 +$$

$$+ \delta^2 (G_0^2 - G^2) = \omega_0^2 G^2 + \delta^2 (G_0 + G)(G_0 - G) =$$

$$= \omega_0^2 G_1 + \frac{R^2}{4L^2} \cdot 2\omega L \quad \frac{2}{\omega C} = \omega_0^2 G^2 + \frac{1}{LC} R^2 =$$

$$= \omega_0^2 (G^2 + R^2) = \omega^2 Z^2$$

$\frac{1}{LC} = \omega_0^2$. hosl qilamiz, chunki . Demak,

$$\frac{\sqrt{\omega_1^2 G + \delta^2 G_0^2}}{Z} = \frac{\omega_0 Z}{Z} = \omega_0.$$

Yuqoridagilarni e’tiborga olib uzil kesil

Yuqoridagilarni e’tiborga olib uzil kesil quyidagi

$$i(t) = \frac{E\omega_0}{\omega_1 Z} e^{-\delta t} \sin(\omega_1 t - \gamma_1) + \frac{E}{Z} \sin(at - \gamma)$$

ni hosil qilamiz.

Differensial tenglamalar sistemasini operasion usul yordamida yechish.

O`zgarmas koeffisiyentli chiziqli differensial tenglamalarni yechishning operasion usullarini bunday tenglamalar sistemalarini yechishga tadbiq qilish mumkin. Farq, shundan iboratki, bitta operator tenglama o`rniga izlanayotgan funksiyalarning tasvir lariga nisbatan chiziqli algebraik operator tenglama lar sistemasi hosil bo`ladi. Bunday berilgan differensial tenglamalar sistemasini oldindan o`zgartirib olishga zarurat qolmaydi, masalan, ularni normal shaklga keltirish zarur emas. Berilgan har qanday sistemani, u qanday berilgan bo`lsa shu ko`rinishda operasion usul yordamida yechish mumkin.

Birinchi tartibli tenglamalar sistemasi berilgan bo`lsin

$$\left.\begin{aligned} x_1'(t) + \sum_{k=1}^{n} a_{1k}\, x_k(t) &= f_1(t), \\ x_2'(t) + \sum_{k=1}^{n} a_{2k}\, x_k(t) &= f_2(t), \\ &\cdot \\ x_n'(t) + \sum_{k=1}^{n} a_{nk}\, x_k(t) &= f_n(t). \end{aligned}\right| \qquad (13)$$

Quyidagi

$$x_k(0) = x_{k_0}, \quad (k = \overline{1, n}) \qquad (14)$$

boshlang'ich shartlarni qanoatlantiruvchi xususiy

insin, bunda

$$L\{f_k\} = F_k(p), \quad L\{x_k(t)\} = X_k(p).$$

Operator tenglamalar sistemasi quyidagi

$$\left.\begin{aligned} pX_1(p) + \sum_{k=1}^{n} a_{1k} X_k(p) &= F_1(p) + x_{10}, \\ pX_2(p) + \sum_{k=1}^{n} a_{2k} X_k(p) &= F_2(p) + x_{20}, \\ &\cdots \\ pX_n(p) + \sum_{k=1}^{n} a_{nk} X_k(p) &= F_n(p) + x_{n0}, \end{aligned}\right\} \qquad (15)$$

ko`rinishda bo`ladi.(15) algebraik chiziqli tenglamalar sistemasini $X_k(p)$ tasvirlarga nisbatan yechib, so`ngra topilgan tasvirlardan $x_k(t)$ orginallarga o`tish kerak, ular birgalikda (13) differensial tenglamalar sistemasining (14) boshlang'ich shartlarni qanoatlantiruvchi xususiy yechimini beradi.

O`zgarmas koeffisiyentli chiziqli differensial tenglamalar sistemalarini operasion hisob usllar yordamida yechishga misollar ko`ramiz:

$$\begin{cases} x' + 3x - 4y = 9e^{2t} \\ 2x + y' - 3y = 3e^{2t} \end{cases}$$

Misol1.

Ushbu

chiziqli differensial tenglamalar sistemasini boshlang'ich shartlarda yeching $x(0) = 2, y(0) = 0$

Yechish. Quyidagiga egamiz.

$$e^{2t} \leftarrow \frac{1}{p-2}$$

$x(t) \leftarrow X(p),\ y(t) \leftarrow Y(p)$ deb olamiz.

Boshlang'ich shartlarga asc $x'(t) \leftarrow pX(p) - 2,\ y'(t) \leftarrow Y(p)$

operatorli tenglamalar sistemasini tuzamiz

$$\begin{cases} pX(p) - 2 + 3X(p) - 4Y(p) = \dfrac{9}{p-2}, \\ 2X(p) + pY(p) - 3Y(p) = \dfrac{3}{p-2} \end{cases}$$

$$\begin{cases} X(p)(p+3) - 4Y(p) - 2 = \dfrac{9}{p-2}, \\ 2X(p) + Y(p)(p-3) = \dfrac{3}{p-2} \end{cases}$$

$$\begin{cases} X(p)(p+3) - 4Y(p) = \dfrac{2p+5}{p-2}, \\ 2X(p) + Y(p)(p-3) = \dfrac{3}{p-2} \end{cases}$$

Bu sitemani X(P) va Y(P) ga nisbatan, yechib quyidagini hosil qilamiz

$$X(p) = \frac{2p^2 - p - 3}{(p^2 - 1)(p - 2)}, \quad Y(p) = -\frac{p + 1}{(p^2 - 1)(p - 2)}$$

Yoki

$$X(p) = \frac{2p - 3}{(p - 1)(p - 2)}, \quad Y(p) = -\frac{1}{(p - 1)(p - 2)}.$$

Topilgan tasvirlarni eng sodda kasrlarga yoyib, quyidagini topamiz

$$X(p) = \frac{1}{p - 1} + \frac{1}{p - 2}, \quad Y(p) = \frac{1}{p - 1} - \frac{1}{p - 2}.$$

Tasvirlar jadvalidan foydalanib:

$$x(t) = e^t + e^{2t},$$
$$y(t) = e^t - e^{2t}.$$

Shaxtada havo oqimini ta’minlovchi **ventilyator** elektr dvigatel orqali harakatga keltiriladi. Ventilyatorning aylanish tezligi o‘zgarganda, havo oqimining qarshiligi ham o‘zgaradi, bu esa **elektr yuritma – mexanik tizim** o‘rtasida bog‘liq jarayonni hosil qiladi. Bu tizimni **differensial tenglamalar sistemasi** orqali ifodalash mumkin.

$$\begin{cases} J\dfrac{d\omega(t)}{dt} + b\omega(t) = M_d(t) - M_h(t) \\ L\dfrac{di(t)}{dt} + Ri(t) = U(t) - k_e\omega(t) \end{cases}$$

Bu yerda

J-ventilyatorning inersiya momenti,

b- mexanik ishqalanish koeffisienti,

$M_d(t) = k_i i(t)$ – dvigatel momenti

$M_h(t) = M_{h_0} \omega^2\ (t)$ -havo qarshiligi momenti (ventilyator yuklamasi),

L,R- dvigatel chulg‘amaning induktivligi va qarshiligi,

$U(t)$ -berilgan kuchlanish,

k_t, k_e – dvigatelning momenti va EYuk koeffitsiyentlari,

$\omega(t)$ -burchak tezlik,

$i(t)$ -tok

$$\begin{cases} J\dfrac{d\omega}{dt} + (b + k_h)\omega = k_t i \\ L\dfrac{di}{dt} + Ri = U(t) - k_e\omega \end{cases}$$

Laplas alamashtrish yordamida yeching:

$\omega(0) = 0, i(0) = 0$

$$\begin{cases} JpW(p) - w(0) + (b + k_h)W(p) = k_t I(p) \\ LpI(p) - i(0) + RI(p) = U(p) - k_e W(p) \end{cases}$$

$U(t) = U_0$ (doimiy kuchlanish $W(p)$ ni toping.

Natijada Ikki tenglamani birlashtiramiz

$$W(p) = \frac{k_t}{(Jp + b + k_h)(Lp + R) + k_t k_e} U(p)$$

Agar $U(t) = U_0$

$$U(p) = \frac{U_0}{s}$$

$$W(p) = \frac{k_t U_0}{s((Jp + b + k_h)(Lp + R) + k_t k_e)}$$

Shundan so’ng Laplas orqali w(t) topiladi.

Adabiyotlar:

1 Dennis G. Zill. Loyola Marymount University. “Advanced engineering mathematics”. Six editions. 2018.

2. Axmedov A.B., Shamsiyev R.B., Shamsiyev D.N., Pirmatov Sh.T. Oliy matematika. Darslik,1 qism. Toshkent, “Fan va texnologiyalar” nashriyoti 2018. -366 б.

3. Xurramov SH. R. Oliy matematika.3-qism. – Toshkent: “Tafakkur” nashriyoti, 2018.

4. www.ziyonet.uz

E’tiboringiz uchun raxmat!