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SAFAR XODJIEV

OLIY MATEMATIKA

(Chiziqli algebra, analitik geometriya va vektor algebrasi)

O’QUV QO’LLANMA

O’ZBEKISTON RESPUBLIKASI
OLIY TAЪЛИМ VA INNOVATSIYALAR VAZIRLIGI

A.PIRIMOV, S.XODJIEV

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Ushbu o’quv qo’llanma, oliy matematika fanining chiziqli algebra, analitik geometriya va vektor algebrasi bo’limlarida, o’qitiladigan mavzularni o’z ichiga olgan bo’lib, texnika universitetlari barcha texnik yo’nalishlari va iqtisod yo’nalishida o’qitiladigan “Oliy matematika” va “Amaliy matematika” fanlari talablari va xususiyatlaridan kelib chiqqan holda, innovatsion ta’lim texnologiyalari hamda axborot-kommunikatsiya texnologiyalariga asoslanib matematik qa’tiyatlik bilan hamda ilg`or tajribalarga ilmiy yondoshib bayon qilingan.

Taqrizchilar:

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fizika - matematika fanlari nomzodi, dotsent, O.J.Xudoyberdiev

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SO’Z BOSHI

O‘zbekiston Respublikasi Prezidenti SH.M.Mirziyoyev 2020 yil ustozlar va murabbiylar bayramiga bag’shlangan tantanali tadbirda, Respublikaning barcha tumah va shaharlarida, jahonga yuz tutgan Qorako’l maktabi tizimida ish olib boradigan maktablar tashkil etish, ularda matematika fanini chuqur o’qitish asosida iqtidorli yoshlarni tayyorlash vazifasini qo’ydi.

Texnika oliy ta’lim muassalarida ham oliy matematika fanini zamon talablari asosida o’qitish asosiy o’rin tutadi, chunki texnik fanlarni talabalar tomonida o’zlashtirishda oliy matematikani o’rni beqiyos.

Iqtidorli va etuk malakali yosh mutaxasislarni tayyorlash mamlakatimiz rivojining jahon miqyosida yqori darajaga chiqishda asosiy o’rin egallaydi. Talabalarning oliy ta’lm o’quv yurtida oladigan nazariy bilimlar natijalarni analiyotga tadbiq qilish malakasini oshirish Respublikani jadal su’ratda rivojlantirish ushun asos bo’ladi. Bu vazifalarni bajardigan ytuk kadrlarni tayyorlash ustozlarga, yanada ulkan vazifalarni yuklaydi va bu vazifalarni bajarish uchun harakat qilish, biz uchun shon sharafga aylanishi zarur.

Qo’llanmada oliy matematika fanining chiziqli algebra, analitik geometriya va vektor algebrasi bo’limlarida, texnika oliy o’quv yurtlarida o’qitiladigan mavzular etarlicha to’liq bayon etilgan.

Shuni mamnuniyat bilan qayd etish kerakki, O’zbekiston Respublikasi Prezidenti SH.M.Mirziyoyevning davlatimiz rivojlanishi uchun amalga oshirayotgan islohotlari, ayniqsa ilm fan rivojini davlat siyosati darajasiga olib chiqqani tahsinga sazovordir.

Prezident SH.M.Mirziyoyev matematika, kimyo, biologiya, geologiya, o’zbek tili va adabiyoti kabi fanlarni jadal rivojlantirish, ularni amaliy tatbiqlariga e’tiborni kuchaytirish kerakligini ma’sullarga tushuntirdilar.

Qarorda matematika fanlarini ta’limning barcha bosqichlarida o‘qitish tizimini yanada takomillashtirish, pedagoglarning samarali mehnatini qo‘llab-quvvatlash, ilmiy tadqiqot ishlarining ko‘lamini kengaytirish va amaliy

ahamiyatini oshirish, xalqaro hamjamiyatlar bilan aloqalarni mustahkamlash ko’zda tutilgan.

Maqsadli dasturni amalga oshirishning ustuvor yo‘nalishlari etib quyidagilar belgilangan.

- ta’lim tashkilotlarida ilg‘or xorij tajribasi asosida matematik tasavvurlarni shakllantirishning zamonaviy pedagogik texnologiyalarini joriy qilish;

- oliy ta’lim muassasalarida matematika fanini o‘qitish sifatini oshirish;

- matematika sohasidagi ilmiy-tadqiqotlarning ishlab chiqarish bilan uzviy bog‘liqligini ta’minlash, amaliy matematikani rivojlantirish va iqtisodiyot tarmoqlaridagi muammolarni modellashtirish asosida matematik yechimlarni ishlab chiqish;

- oliy ta’lim muassasalarida matematika fanini o‘qitishda chet eldagi universitet hamda ilmiy tashkilotlar bilan aloqalarni rivojlantirish, o’zaro kadr almashishni tashkil qilish.

Matematika fani va unig rivojlanish tarixiga bir nazar

Matematika yunoncha **«ilm», «fan»** so’zlaridan olingan bo’lib - moddiy dunyoning miqdoriy munosabatlari va fazoviy shakllari haqidagi fandir. Qadimgi Misr va Bobilda arifmetika, geometriya fan sifatida shakllandi. Qadimgi Yunonistonda matematik faktlar mantiqiy tartibga solindi. Qadimgi yunon mathematicising asosiy yutug’i matematikaga isbot tushunchasining kiritilishi bo’ldi.

Miloddan avvalgi III-asrdan boshlab ellinizim davri boshlandi, ilmiy tadqiqotlar markazi Shimoliy Afrika, xususan, Iskandariya va Sirako’zaga ko’chdi.

Qadim Yunoniston va ellinizim davri bilan bir vaqtda qadimgi Xitoy va Hindistonda ham matematika rivojlandi.

O’rta asrlarda fan, jumladan matematika rivojlandi, uning taraqqiyotida yangi davr boshlandi. IX - asrga kelib arab tilida ijod qilgan matematiklar matematik-

astronomik jadvallar tuzdilar, algebra, geometriya, trigonometriya sohasida qator yutuqlarga erishdilar.

Xorazmiyning «Al-jabr val-muqobola» asari algebra faniga asos soldi. Xorazmiy bu asarida birinchi marta algebraik almashtirishlar, kvadrat tenglamalarni echish va boshqa algebraik masalalarni sistematik bayon qildi. Matematikada algoritmlarni qo'llash, geometriyaning algebraga, algebraning geometriyaga tadbiqi Xorazmiy ishlaridan so`ng keng tarqaldi.

X-asrda Abu Rayxon Beruniy, Ibn Al-Haysan, Umar Hayyom va boshqalar uchinchi va to'rtinchi darajali tenglamalardan geometrik masalalarni echishda foydalanadilar.

Umar Nayyom kub tenglamalarni geometrik yo'l bilan mukammal tekshirdi va u matematikaga oid 70 dan ortiq ilmiy asarlar yozib qoldirgan.

Bu davrda astronomiya bilan trigonometriya ham turli faktlar bilan boyidi. O'rta va yaqin sharqda trigonometrik funksiyalardan keng foydalandilar, yuksak aniqlikda matematik jadvallar tuzildi va trigonometriyaga oid formulalar isbot qilindi. Masalan Abul Vafo jadvalida sinusning qiymatlari 10 minut oraliq bilan

10^{-4} aniqliqda hisoblangan. Nasriddin Tusiy sferik uchburchaklarni yechish masalasini to'la hal qildi.

Matematikaning rivojlanishida Ulug'bek ilmiy maktabining xizmatlari katta bo'ldi. Ulug'bek, Qozizoda Rumiy, Ali Qushchi, Koshiy va boshqa olimlar algebra, geometriya, trigonometriya, astronomiya sohalarida yuksak natijalarga erishdilar. Masalan: $x = \sin 10$ ni hisoblash uchun Koshiy $x^3-45x+0{,}7850393433644006=0$ tenglamani tuzadi va ketma-ket yaqinlashish usuli bilan echib unung

$$\sin 10^0 = 0{,}017452406437283571$$

qiymatini topadi. Shuningdek u aylanaga ichki va tashqi chizilgan muntazam $3\bullet 228$ burchakli ko'pburchakning perimetrini topib π sonini 17 xona aniqlikda hisobladi.

Samarqand matematiklari Umar-Hayyom - Nyuton binomi formulasi, irrasional sonlar nazariyasi, tenglamlarni taqribiy echish usullari va sonlar nazariyasining ayrim masalalarini ishlab chiqdilar.

Matematikaning keyingi rivoji 12-16 asrlarda (Uygonish davri) Italiyada davom etdi, sharq fani va madaniyati yodgorliklari, jumladan matematik qo'lyozmalar Evropada o'rganila boshladi. 16-17va xususan 18 asrlarda va undan keyin ham Yevropa mamlakatlarida matematika jadal sur'atlar bilan rivojlandi. Matematika g`oyat kengayib va tarmoqlanib ketdi.

Asrlar davomida yuzlab-minglab ulug' matematiklarning qilgan buyuk xizmatlari oqibatida hozirgi zamonning g'oyat tarmoqlangan va beqiyos yutuqlarga erishgan, minglab amaliy va nazariy masalalarni hal qilgan va qilayotgan buyuk matematika fani yaratildi. U hamon rivojlanishda, taraqqiyotda. 20-asrga kelib bizning O'zbekistonimizda ham boshqalar qatori hozirgi zamon matematikasi yanada yangidan-yangi yutuqlarga erishdi. Matematikaning deyarli hamma tarmoqlari buyicha O`zbekistonimizda ham yangidan-yangi yutuqlarga erishildi.

Matematika, fizika, kimyo, mexanika, biologiya va boshqa fanlar hamda sohalarda hodisalarni o'rganishda uchraydigan kattaliklar, ikki xil: skalyar va vector miqdorlardir. Skalalyar kattaliklar deb ataladigan kattaliklar sinfi mavjud bo'lib, ularni xarakterlash uchun bu kattaliklarni son qiymatlarini ko'rsatish yetarlidir. Bular, masalan, hajm, massa, zichlik, harorat va boshqalardir. Lekin shunday kattaliklar mavjudki, ular faqat son qiymatlari bilangina emas, balki yo'nalishi bilan ham xarakrerlanadi. Ular yo'nalgan kattaliklar yoki vektor kattaliklar deb ataladi. Harakat tezligi, magnit yoki elektr maydonning kuchlanganligi va boshqa kattaliklar shunga misol bo'ladi.

O'quv qo'llanma kamchiliklardan xoli bo'Imasligi mumkin. Uni yanada mukammallashtirishga qaratilgan tanqidiy fikr va mulohazalarini bildiradigan hamkasblarga oldindan o'z tashakkurimizni izhor etamiz.

Ulug' matematiklarimiz

Extimollar nazariyasi va matematik statistika fani fidoyi olimlarimiz T.A. Sarimsoqov va S.X.Sirojiddinovlar va ularning shogirdlari tomonidan yanada rivojlantirilmoqda.

Akademik T.A. Sarimsoqov ikkinchi jahon urushi qatnashchisi, harbiy meteorolog (1941-1942). Harbiy xizmat davrida “Holatlari sanoqli bo`lgan Markov zanjirlari haqida” doktorlik dissertatsiyasini yo’qlagan. Uning dastlabki ishlari extimollar nazariyasi metodlarini klassik analiz qilish masalalariga tatbiq etishga bag’ishlangan. Uning katta siklni tashkil qiladigan ishlarida Markov zanjirlari tekshirilgan. U birinchi marta matritsalar metodini holatlari sanoqli bo’lgan Markov zanjirlariga qo`llagan, matritsalar metodini xarakteristik funktsiyalar metodi bilan birlashtirish asosida extimollar nazariyasining katta sonlar qonuni, markaziy limit teorema, takroriy logarifm qonuni va boshqa teoremalarini sanoqli va uzluksiz holatli Markov zanjirlariga yoygan.

T.A.Sarimsoqovning bir qator ishlari bir jinsli bo’lmagan Markov zanjirlariga oid. Xususan N.A.Kolmogorov va V.I.Romonovskiy usullarining sinteziga oid ishlari muxim. Uning qator ishlari o’rta va Old Osiyo sinoptik jarayonlarini o’rganish masalalarida extimollik sxemalarini qo’llanishga bag’ishlangan. U bu ishlari uchun Davlat mukofoti sovrindori. Uning o’zi yaratgan va rivojlantirgan topologik yarim maydonlar nazariyasidir. Bu ishlar natijasida shu soha bo’yicha Toshkent maktabiga asos solindi. U Beruniy mukofoti sovrindori hamdir.

Ulug’ matematiklarimizdan yana biri Sirojiddinov Sa'di Hasanovichdir. Uning dastlabki ishlari ko’p o’zgaruvchili klassik ko’phadlar xossalariga bag’ishlangan. S.H.Sirojiddinovning Markov zanjiri uchun teoremalarining umumlashtirish va asimtotik yoyilmalar sohasidagi tatdiqotlari shu sohada birinchi ishdir.

S.H.Sirojiddinov extimollar nazariyasi sohasidagi yutuqlarni matematik sintaktikaga tatbiq qildi. S.H.Sirojiddinov va uning shogirdlari o’zaro bog’liq bo’lmagan va Markov zanjirini tashkil qiluvchi tasodifiy miqdorlar va tasodifiy vektorlar yig`indisi uchun limit teoremalariga doir muxim masalalarni hal qilar; shu masalalar bilan bog`lik tekis baholashlariga oid tatqiqotlari katta tadbikiy ahamiyatga ega. S.H.Sirojiddinov ommaviy xizmat ko`rsatish nazariyasi, sintaktikaning turli sohalari, statsionar jarayonlar nazariyasi va boshqa sohalarda ham keng ko`lamda tatqiqot olib bordi.

S.H.Sirojiddinov Bernulli nomidagi xalqaro matematik sintaktika va extimollar nazariyasi jamiyati (Gollandiya) ning haqiqiy a'zosi edi. Beruniy mukofoti sovrindori .

Albatta yana bir qancha atoqli o`zbek matematiklarining: masalan; Olimov Shavkat, Ayupov Shavkat, Sa'dullaev Azimboy va Hojiev Javat kabilarni aytish mumkin va ularning ishlarini ko`rsatib o`tish mumkin edi.

Rejaga ko`ra biz birinchi yarim yillikda quyidagilarni o`rganamiz. Endi birinchi yarim yillikda birinchi kurs talabalariga o`tiladigan darslar mavzusiga to’xtalmoqchimiz.

Birinchi navbatda bu albatta chiziqli algebradir. Matematikaning bu bo`limida chiziqli fazolar va ularning chiziqli akslantirishlari o`rganiladi. Chiziqli tenglamalarni o`rganish jarayonida qo`llana boshlagan aniqlovchi (determinant), vektorlar, matritsalar kabi tushunchalar matematikada o`zaro qo`shish va skalyarga ko`paytirish mumkin bo`lgan ob'ektlar alohida o`rin tutishini anglashga, ularni boshqa konkrekt xossalaridan ajralgan xolda o`rganishga tenglamalarini kanonik (eng sodda) ko`rinishga keltirish masalasi chiziqli algebra masalasidan iborat ekanligi aniqlangach, chiziqli algebra ko`p o`lchovli fazo analitik geometriyasi bilan qo`shilib ketdi va chiziqli, bichiziqli, kvadratik formalar, chiziqli almashtirish va akslantirish: evklid fazosi: proektiv fazo tushunchalari bilan boyidi.

Analitik geometriya-matematikaning bir bo`limi; unda geometrik obrazlar (nuqta, chiziq, sirt va hakozolar) koordinatalar metodi asosida algebraik vositalar yordamida o`rganiladi. Bu erda chiziqlar ularning tenglamalari vositasida o`rganiladi. Masalan; radusi r, markazi koordinatalar boshida bo`lgan aylana $x^2+y^2=r^2$ tenglama bilan, abstsissalar o’qi a burchak tashkil qilib ordinata uqidan OB=b kesma ajratgan tug`ri chiziq esa y=kx+b, k = tga tenglama bilan aniqlanadi.

Uch o'lchovli fazoda uchta koordinatani bog’lovchi F(x,y,z) = 0 tenglama biror sirtni aniqlaydi. Masalan birinchi darajali tenglama to’gri burchakli dekart koordinatalarda biror tekislikni aniqlaydi. Radiusi r, markazi (a,b,c) nuqtada bo’lgan sferik sirt (x-a)2+(y-b)2+(z-c)2=2 tenglama bilan aniqlanadi va hakozo.

Demak, analitik geometriyada geometrik obrazlar koordinatalar vositasida algebraik yo’l bilan tekshiriladi.

Bu fanning fan sifatida shakllanishida R. Dekart va P. Ferma kabi frantsuz olimlarining xizmatlari kattadir. Fan va texnika taraqqiyotida katta ahamiyatga ega bologna differentsial va integral hisob Nyuton va Leybnitslar tomonidan 17-asrda yaratildi. Differentsial hisob matematikaning hosilalar va differentsiallarni hisoblash, xossalarni o`rganish hamda hosila va differentsiallarni funktsiyalarni tekshirishga tatbiq qilish bilan shug`ullanadi gan bo`lim.

Matematikaning bu bo`limi matematik analizning muhum bo`limidir. Egri chiziqqa urunma o`tkazish, o`tilgan yo`l vaqtning funktsiyasi shaklida berilganda tezlik va tezlanishlarni topish, funktsiyalar qiymatlarini oldindan talab qilingan aniqlikda hisoblash hamda amaliy ishlar uchun beqiyos muhim bo`lgan turli funktsiyalar jadvallarini tuzish, funktsiyalarning o`sish va kamayish oraliqlarini, funktsiya grafigining burilish nuqtalarini topish va bundan nuqtalarda uning qiymatlarini topish (maksimum va minimum qiymatlar), funktsiyaning eng katta va eng kichik qiytmalarini topish, funktsiya grafigining qavariqlik va botiqlik oraliqlarini aniqlash, ko’p o’zaruvchili funksiya tushunchasi, unga oid barcha ma’lumotlar va hokazo masalalar shu birinchi yarim yillikda o`tilishi ko`zda tutilgan.

K I R I SH

Oliy ta’limda “Oliy matematika” fanining o’qitish usullari va ularni dars jarayonida samarali qo’llash haqida qisqa ma’lumot.

Oliy ta’limda matematika fanini modulli o‘qitishning maqsadi, oliy ta’lim talabalarini zamonaviy o‘qitish uchun metodlarni tanlash, tahlil etish va ularni Modul bo‘yicha bilimi, ko‘nikmasi, malakasini yuqori darajaga ko‘tarishda modulli o‘qitishning quyidagi intrefaol usullardan foydalanish maqsadga muvofiq deb hisoblaymiz.

AQLIY HUJUM METODI

“Aqliy hujum” metodi- biror muammo bo‘yicha ta’lim oluvchilar tomonidan bildirilgan erkin fikr va mulohazalarni to‘plab, ular orqali ma’lum bir yechimga kelinadigan metoddir. “Aqliy hujum” metodining yozma va og‘zaki shakllari mavjud. Og‘zaki shaklida ta’lim beruvchi tomonidan berilgan savolga ta’lim oluvchilarning har biri uz fikrini og‘zaki bildiradi. Ta’lim oluvchilar o‘z javoblarini aniq va qisqa tarzda bayon etadilar. Yozma shaklida esa berilgan savolga ta’lim oluvchilar o‘z javoblarini qog‘oz kartochkalarga qiska va barchaga ko‘rinarli tarzda yozadilar. Javoblar doskaga (magnitlar yordamida) yoki “pinbord” doskasiga (ignalar yordamida) mahkamlanadi. “Aqliy hujum” metodining yozma shaklida javoblarni ma’lum belgilar bo‘yicha guruhlab chiqish imkoniyati mavjuddir. Ushbu metod to‘g‘ri va ijobiy qo‘llanilganda shaxsni erkin, ijodiy va nostandart fikrlashga o‘rgatadi.

“Aqliy hujum” metodidan foydalanilganda ta’lim oluvchilarning barchasini jalb etish imkoniyati bo‘ladi, shu jumladan ta’lim oluvchilarda muloqot qilish va munozara olib borish madaniyati shakllanadi. Ta’lim oluvchilar o‘z fikrini faqat og‘zaki emas, balki yozma ravishda bayon etish mahorati, mantiqiy va tizimli fikr yuritish ko‘nikmasi rivojlanadi. Bildirilgan fikrlar baholanmasligi ta’lim oluvchilarda turli g‘oyalar shakllanishiga olib keladi. Bu metod ta’lim oluvchilarda ijodiy tafakkurni rivojlantirish uchun xizmat qiladi.

“Aqliy hujum” metodi ta’lim beruvchi tomonidan qo‘yilgan maqsadga qarab amalga oshiriladi:

1. Ta’lim oluvchilarning boshlang‘ich bilimlarini aniqlash maqsad qilib qo‘yilganda, bu metod darsning mavzuga kirish qismida amalga oshiriladi.

2. Mavzuni takrorlash yoki bir mavzuni keyingi mavzu bilan bog‘lash maqsad qilib qo‘yilganda –yangi mavzuga o‘tish qismida amalga oshiriladi.

3. O‘tilgan mavzuni mustahkamlash maqsad qilib qo‘yilganda mavzudan so‘ng, darsning mustahkamlash qismida amalga oshiriladi.

“KICHIK GURUKLARDA ISHLASH” METODI - ta’lim oluvchilarni faollashtirish maksadida ularni kichik guruhlarga ajratgan holda o‘quv materialini o‘rganish yoki berilgan topshiriqni bajarishga qaratilgan darsdagi ijodiy ish.

Ushbu metod qo‘llanilganda ta’lim oluvchi kichik guruhlarda ishlab, darsda faol ishtirok etish huquqiga, boshlovchi rolida bo‘lishga, bir-biridan o‘rganishga va turli nuqtai- nazarlarni qadrlash imkoniga ega bo‘ladi.

“Kichik guruhlarda ishlash” metodik qo‘llanilganda ta’lim beruvchi boshqa interfaol metodlarga qaraganda tejash imkoniyatiga ega buladi. Chunki ta’lim beruvchi bir ta’lim oluvchilarni mavzuga jalb etaoladi va baholay oladi.

“BAHS-MUNOZARA” METODI – biror mavzu bo‘yicha ta’lim oluvchilar bilan uzaro bahs, fikr almashinuv tarzida o‘tkaziladigan o‘kitish metodidir.

Har handay mavzu va muammolar mavjud bilimlar va tajribalar asosida muhokama qilinishi nazarda tutilgan holda ushbu metod qo‘llaniladi. Baxs – munozarani boshqarib borish vazifasini ta’lim oluvchilarning biriga topshirishi yoki ta’lim beruvchining o‘zi olib borishi mumkin. Bahs- munozarani erkin holatda olib borish va har bir ta’lim oluvchini munozaraga jalb etishga xarakat qilish lozim. Ushbu metod olib borilayotganda ta’lim oluvchilar orasida paydo bo‘ladigan nizolarni darhol bartaraf etishga xarakat qilish kerak.

“Bahs-munozara” metodini o‘tkazishda quyidagi qoidalarga amal qilish kerak:

U barcha ta’lim oluvchilar ishtirok etishi uchun imkoniyat yaratish;

U “o‘ng qo‘l” qoidasi (qo‘lini ko‘tarib, ruxsat olgandan sung so‘zlash)ga rioya qilish;

U fikr mulohzalarni tinglash madaniyati;

U bildirilgan fikr mulohzalarni takrorlanmasligi;

U bir-birlariga o‘zaro hurmat.

LOYIHA” METODI

“loyiha” metodi – bu ta’lim oluvchilarning individual yoki guruhlarda belgilangan va kun davomida, belgilangan mavzu bo‘yicha axborot yig‘ish, tadqiq o‘takazish va amalga oshirish ishlarini olib borishidir. Bu metodda ta’lim oluvchilar rejalashtirish, qaror qabul qilish, amalga oshirish, tekshirish va xulosa chiqarish va natijalarni baholash jarayonlarida ishtirok etadilar. Loyiha ishlab chiqish yakka tartibda yoki guruh bo‘lishi mumkin, lekin har bir loyiha o‘quv guruhining birgalikdagi faoliyatining muvofiklashtirilgan natijasidir.

Loyiha o‘rganishga xizmat qilishi, nazariy bilimlarni amaliyotga tadbiq etishi, ta’lim oluvchilar tomonidan mustaqil rejalashtirish, tashkillashtirish va amalga oshirish imkoniyatini yarata oladigan bo‘lishi kerak. Quyidagi chizmada “Loyiha” metodining bosqichlari keltirilgan.

“KEYS-STADI” METODI.

“Keys-stadi” - inglizcha so ‘z bo ‘lib, (“case”- aniq vaziyat, hodisa, “stadi’ - o‘rganmoq, tahlil qilmoq) aniq vaziyatlarni o‘rganish, taxlil qilish asosida o‘qitishni amalga oshirishga xaratilgan metod xisoblanadi. Mazkur metod dastlab 1921-yil Garvard universitetida amaliy vaziyatlardan iqtisodiy boshqaruv fanlarini o‘rganishda foydalanish tartibida qo‘llanilgan. Keysda ochiq axborotlardan yok aniq voqea-hodisadan vaziyat sifatida tahlil uchun foydalanish mumkin. Keys xarakatlari o‘z ichiga quyidagilarni qamrab oladi: Kim (Who), Kachon (When), Kaerda (Where), Nima uchun (Why), Kanday, qanaqa(How), Nima-natija (What).

“FSMU” METODI.

Texnologiyaning maqsadi: Mazkur texnologiya ishtirokchilardagi umumiy fikrlardan xususiy xulosalar chiqarish, taqqoslash, qiyoslash, o‘zlashtirish, xulosalash, shuningdek, mustaqil ijodiy fikrlash ko‘nikmalarini shakllantirishga xizmat qiladi. Mazkur texnologiyadan ma’ruza mashg‘ulotlarida, mustahkamlashda,

o‘tilgan mavzuni so‘rashda, uyga vazifa berishda hamda amaliy mashg‘ulot natijalarini tahlil etishda foydalanish tavsiya etiladi.

“INSERT” METODI

Metodning maqsadi: Mazkur metod o‘quvchilarda yangi axborotlar tizimini qabul qilish va bilimlarni o‘zlashtirilishini yengillashtirish maqsadida qo‘llaniladi, shuningdek, bu metod o‘quvchilar uchun xotira mashqi vazifasini ham o‘taydi.

XULOSALASH (REZYUME, VEER) METODI.

Metodning maqsadi - murakkab, ko‘p tarmoqli, mumkin qadar, muammoli xarakteridagi mavzularni o‘rganishga qaratilgan. Metodning mohiyati shundan iboratki, bunda mavzuning turli tarmoqlari bo‘yicha bir xil axborot beriladi va ayni paytda, ularning har biri alohida aspektlarda muhokama etiladi. Masalan, muammo ijobiy va salbiy tomonlari, afzallik, fazilat va kamchiliklari, foyda va zararlari bo‘yicha o‘rganiladi. Bu interfaol metod tanqidiy, tahliliy, aniq mantikiy fikrlashni muvaffaqiyatli rivojlantirishga hamda o‘quvchilarning mustaqil g‘oyalari, fikrlarini yozma va og‘zaki shaklda tizimli bayon etish, himoya qilishga. imkoniyat yaratadi. “Xulosalash” metodidan ma’ruza mashg‘ulotlarida individual va juftliklardagi ish shaklida, amaliy va seminar mashg‘ulotlarida kichik guruhlardagi ish shaklida mavzu yuzasidan bilimlarni mustahkamlash, tahlil qilish va taqqoslash maqsadida foydalanish mumkin.

I Bob CHIZIQLI ALGEBRA ELEMENTLARI

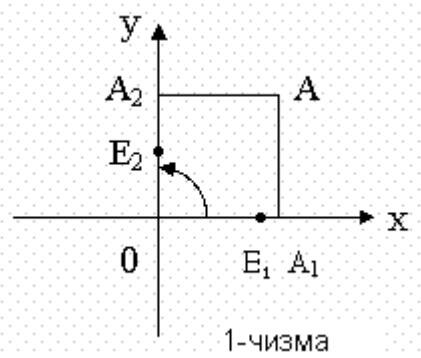
1.1-§ TEKISLIKDA KOORDINATALAR METODI.

1. To’g’ri burchakli Dekart koordinatalar sistemasi.
2. Kesma uzunligi.
3. Kesmani berilgan nisbatda bo’lish.

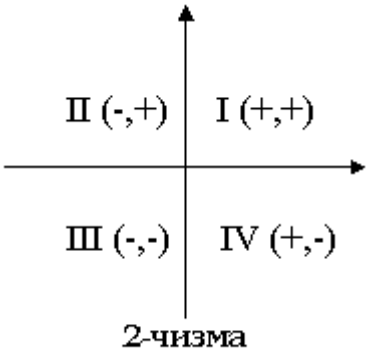
Tekislikda o’zaro perpendikulyar ikkita to’g’ri chiziqlarni qaraylik. O ularning kesishish nuqtasi bo’lsin. Bu to’g’ri chiziqlarda uzunlik bo’yicha $OE_1 = OE_2$ shartni qanoatlantiruvchi E_1 va E_2 nuqtalarni belgilaymiz. OE_1 va OE_2 to’g’ri chiziqlarni absissalar va ordinatalar o’qi deb ataladi va mos ravishda Ox va Oy orqali belgilanadi.

O nuqtani esa koordinatalar boshi deyiladi. O nuqta o’qlarning har birini ikkita yarim o’qlarga ajratadi. Yarim o’qlardan birini shartli ravishda musbat, ikkinchisini esa manfiy deb ataymiz. chizmada musbat yarim o’q uchiga strelka qo’yamiz. $E_1 \in Ox$ nuqta $E_2 \in Oy$ nuqtaga O markaz atrofida soat strelkasiga teskari ravishda 90° burish orqali o’tadi. Tekislikning har bir A nuqtasiga ikkita x, y sonlarini mos keltiramiz.

A nuqtadan ordinata o’qi Oy ga parallel to’g’ri chiziq o’tkazib, uning Oy o’qi bilan kesishgan nuqtasini A_2 orqali belgilaymiz. $A_1 \in Ox$ bo’lib, OA_1 kesma uzunligi x orqali belgilaymiz. A_1 nuqta musbat yarim o’qda yotsa x musbat son, manfiy yarim o’qda yotganda esa manfiy son bo’ladi. O nuqta bilan ustma-ust tushsa, 0 (nol) son bo’ladi. $A_2 \in Oy$ bo’lib, OA_2 kesma uzunligini y orqali belgilaymiz. y son x kabi musbat, manfiy yoki nol sonlar bo’lishi mumkin. x-ni A nuqtaning absissasi, y-ni esa ordinatasi deyiladi. x, y sonlar juftini A nuqtaning koordinatalari deyiladi va $A(x,y)$ kabi belgilanadi. $A_1(x,0)$, $A_2(0,y)$, $O(0,0)$ koordinatalarga ega. Koordinata o’qlari kesishib, tekislikni to’rtta kvadratlarga ajratadi. Kvadratlardagi



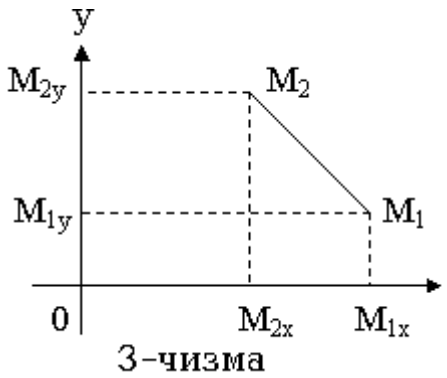
1-чизма



2-чизма

ixtiyoriy nuqta koordinatalarining ishorasi 2-chizmada tasvirlangan.

x, y haqiqiy sonlar jufti (x, y) tartibda berilgan bo'lsa, Oxy tekislikda birdan bir A(x,y) nuqta mos keladi. chizmada A nuqtani ko'rsatish uchun Ox o'qda $A_1(x,0)$, Oy o'qda esa $A_2(0,y)$ nuqtalar belgilanadi. A_1 nuqtadan Oy o'qqa, A_2 nuqtadan Ox o'qqa parallel to'g'ri chiziqlar o'tkazamiz. Ularning kesishish nuqtasi izlangan A(x,y) nuqtadan iboratdir.



3-чизма

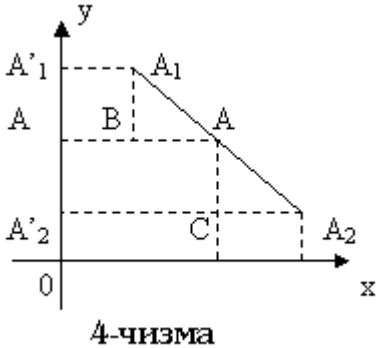
x>0, y>0 bo'lsa A∈I, x<0, y>0 bo'lsa A∈II,

x<0, y<0 bo'lsa A∈III x>0, y<0 bo'lsa A∈IV.

Tekislikda $M_1(x_1,y_1)$ va $M_2(x_2,y_2)$ nuqtalar berilgan bo'lsin. M_1 va M_2 nuqtalar orasidagi masofa deb M_1M_2 kesma uzunligiga aytiladi. M_1M_2 kesma uzunligini shu nuqtalarning koordinatalari orqali aniqlaylik. $x_1 \neq x_2$, $y_1 \neq y_2$ bo'lsin. M_1 va M_2 nuqtalardan koordinata o'qlariga parallel to'g'ri chiziqlar o'tkazamiz. M_1M_{1y} va M_2M_{2x} to'g'ri chiziqlar M nuqtada kesishadi. M_2MM_1 uchburchak to'g'ri burchakli, MM_1 kesma uzunligi $|x_1 - x_2|$ ga, MM_2 kesma uzunligi esa $|y_1 - y_2|$ ga teng. Pifagor teoremasini to'g'ri burchakli M_2MM_1 uchburchakka qo'llash orqali $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$ ifodani hosil qilamiz. Bunda d M_1 va M_2 nuqtalar orasidagi masofa yoki M_1M_2 kesma uzunligini ifoda etadi.

Shunday qilib, $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ (1)

ikki nuqta orasidagi masofaning formulasidir. M_1 va M_2 nuqtalar ustma-ust tushsa $d = 0$. Endi Oxy tekislikda A_1A_2 ikkita turli nuqtalar berilgan bo'lsin. A nuqta A_1A_2 kesmaga tegishli bo'lib, Uni $\lambda_1:\lambda_2$ nisbatda ajratsin. A nuqtaning x, y koordinatalarini A_1 va A_2 nuqtalarning koordinatalari orqali ifodalash talab qilingan bo'lsin. A_1A_2 kesma Ox o'qqa parallel bo'lmasin. A_1, A, A_2 nuqtalarni Oy o'qqa proeksiyalaylik. A'_1, A', A'_2 nuqtalar Oy o'qdagi proeksiyalar bo'lsin. A_1BA va ACA_2 uchburchaklarning



4-чизма

o’xshashligidan (mos burchaklari o’zaro teng)

$$\frac{A_1A}{A_1A_2}=\frac{A'_1A'}{A'A'_2}=\frac{\lambda_1}{\lambda_2} \qquad (2)$$

tengliklarga ega bo’lamiz. $A_1(x_1,y_1)$, $A_2(x_2,y_2)$, $A(x,y)$ nuqtalarning proeksiyalari $A'_1(0,y_1)$, $A'_2(0,y_2)$, $A'(0,y)$ koordinatalarga ega bo’lib,

$$A'_1A'=|y_1-y|,\ A'A'_2=|y-y_2| \qquad (3)$$

(2) va (3) tengliklardan $\frac{|y_1-y|}{|y-y_2|}=\frac{\lambda_1}{\lambda_2}$ (4) kelib chiqadi.

A’ nuqtaning A'_1, $A'_2\in Oy$ nuqtalarning orasida yotishidan y_2-y *va* y_1-y sonlarning bir hil ishorali ekanligi kelib chiqadi. Bundan

$$\frac{|y_1-y|}{|y-y_2|}=\frac{y_1-y}{y-y_2}=\frac{\lambda_1}{\lambda_2} \qquad (5)$$

Ko’ramizki, $y=\frac{\lambda_2y_1+\lambda_1y_2}{\lambda_1+\lambda_2}$. (6)

Yuqoridagi kabi mulohazalarni o’tkazib, A nuqtaning absissasi uchun

$$x=\frac{\lambda_2x_1+\lambda_1x_2}{\lambda_1+\lambda_2} \qquad (7)$$

formulani keltirib chiqaramiz. Agar $\lambda_1:\lambda_2$ ni λ bilan belgilasak,

$$y=\frac{y_1+\lambda y_2}{1+\lambda};\ x=\frac{x_1+\lambda x_2}{1+\lambda} \qquad (8)$$

formulalarga ega bo’lamiz.

(8) da $\lambda\neq1$ shartni bajarilishini talab qilamiz. aks holda A va A_2 nuqtalar ustma-ust tushadi. Agar A nuqta A_1A_2 kesmaning o’rtasida yotsa, $\lambda=1$ bo’lib,

$$y=\frac{y_1+y_2}{2};\quad x=\frac{x_1+x_2}{2} \qquad (9)$$

formula kelib chiqadi.

Agar A nuqta A_1A_2 kesmaning ichki nuqtasi bo’lsa, $\lambda>0$. A_1A_2 to’g’ri chiziqning A_1A_2 kesmasidan tashqaridagi barcha nuqtalar uchun $\lambda<0$ bo’ladi.

Misol: Uchlari A(1,1), B(5,4), S(13,6) orqali berilgan ABS uchburchak A burchak bissektrisasining BE tomon bilan kesishish nuqtasi aniqlansin.

Yechish: λ>0 va $\lambda = \frac{|BD|}{|DC|} = \frac{d(AB)}{d(AC)}$; $d(AB) = \sqrt{16+9} = 5; d(AC) = \sqrt{144+25} = 13.$

$x = \frac{5+5/13\cdot 13}{1+5/13} = \frac{130}{18} = 7\frac{2}{9};$ $y = \frac{4+5/13\cdot 6}{1+5/13} = \frac{82}{18} = 4\frac{5}{9}.$ *Жавоб*: $\left(7\frac{2}{9}, 4\frac{5}{9}\right)$

1.2-§ Aniqlovchilar (determinantlar) va ularning xossalari.

1.2.1. Ikkinchi tartibli determinant tushunchasi

Berilgan to`rtta sondan iborat qo`yidagi

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

jadval 2-tartibli kvadrat matrisa deb ataladi. Bunday matrisa ikkita satr va ikkita ustunga ega. Bu matrisani tuzuvchi sonlar ikkita indeksli, masalan a_{il} $(i, j=1,2)$ harf bilan belgilanadi. Bu yerda i indeks maskur son turgan matrisaning satr nomerini ko`rsatsa, $j$ indeks esa ustun nomerini bildiradi. $a_{11}$, $a_{12}$, $a_{21}$, $a_{22}$ sonlar matritsaning elimentlari deb ataladi. ***Matrisaga mos*** Ikkinchi tartibli determinant deb $a_{11}\ a_{22} - a_{12}\ a_{21}$ songa aytiladi va u qo`yidagicha belgilanadi.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

Bu determinant ikkita a_{11}, a_{12} va a_{21}, a_{22} satr elementlari va ikkita a_{11}, a_{21} va a_{12}, a_{22} ustun elimentlaridan iborat a_{11}, a_{22} elementlarga bosh diagonal elementlari, a_{12}, a_{21} lar esa yordamchi diagonal elementlari deyiladi.

Ta`rif. Quyidagi

$a_{11}\ a_{22}\ a_{33} + a_{12}\ a_{23}\ a_{31} + a_{21}\ a_{32}\ a_{13} - a_{13}\ a_{22}\ a_{31} - a_{32}\ a_{23}\ a_{11} - a_{12}\ a_{21}\ a_{33}$

songa ***uchinchi tartibli aniqlovchi*** deyiladi va

$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ kabi belgilanadi. Ta'rifdan $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\ a_{22}\ a_{33} +$

$+ a_{12}\ a_{23}\ a_{31} + a_{21}\ a_{32}\ a_{13} - a_{13}\ a_{22}\ a_{31} - a_{32}\ a_{23}\ a_{11} - a_{12}\ a_{21}\ a_{33}$ ekanligi kelib chiqadi.

1.2.2. Aniqlovchilar (determinantlar) va ularning xossalari.

1-xossa. Agar aniqlovchining yo`llarini shu nomerli ustunlari bilan almashtirsak aniqlovchining qiymati o`zgarmaydi, ya'ni

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Bu xossa aniqlovchi yo`l va ustunlari teng xuquqli (sof matematik ma'noda) ekanini ko`rsatadi.

2-xossa. Aniqlovchining yo`li yoki ikkita ustuni o`rinlarini almashtirish, uni -1 ga ko`paytirish bilan teng kuchlidir.

3-xossa. Agar aniqlovchining ikkita ustuni (ikkita yuli) bir xil bo`lsa, bu aniqlovchi nolga tengdir.

4-xossa. Aniqlovchining biror yo`l yoki ustun unsurlarini ixtiyoriy k songa ko`paytirish, aniqlovchini shu songa ko`paytirish bilan barobardir.

5-xossa. Agar aniqlovchining biror yo`l yoki ustun unsurlari nollardan iborat bo`lsa, bu aniqlovchi nolga tengdir (bu xossa oldingisidan k=0 da kelib chiqadi).

6-xossa. Agar aniqlovchining ikkita yo`l yoki ikkita ustun unsurlari (proportsional) mutanosib bo`lsa,bunday aniqlovchi nolga tengdir.

7-xossa. Quyidagi xossa o’rinlidir

$$\begin{vmatrix} a_1' + a_1'' & b_1 & c_1 \\ a_2' + a_2'' & b_2 & c_2 \\ a_3' + a_3'' & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} a_1'' & b_1 & c_1 \\ a_2'' & b_2 & c_2 \\ a_3'' & b_3 & c_3 \end{vmatrix}$$

8-xossa. Agar aniqlovchining biror yo`l yoki ustun unsurlariga shu aniqlovchining boshqa yo`l yoki ustun unsurlarini biror noldan farqli songa ko`paytirilganini qo`shsak, aniqlovchi o`zgarmaydi

1.2.3. Algebraik to`ldiruvchi va minorlar

Δ aniqlovchini qo`yidagicha yozamiz:

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c - a_2b_1c_3 - a_1b_3c_2 =$$

$$= a_1(b_2c_3 - b_3c_2) + a_2(b_3c_1 - b_1c_3) + a_3(b_1c_2 - b_2c_1) = a_1A_1 + a_2A_2 + a_3A_3.$$

Bu erda A_1 ,A_2, A_3 lar mos ravishda a_1,a_2 , a_3 larning algebraik to`ldiruvchisi deyiladi.

9-xossa. Aniqlovchi uning biror yo`l yoki ustun unsurlarini ularga mos algebraik to`ldiruvchilar bilan ko`paytmalarining yig`indisiga teng.

Aniqlovchining biror unsurlarining minori deb, shu unsur turgan yo`l va ustun unsurlarini o`chirgandan so`ng hosil bo`lgan aniqlovchiga aytiladi.

Misol: Δ aniqlovchi a_1 unsurning minori $\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$ aniqlovchi bo`ladi.

Aniqlovchining biror unsurlari algebraik to`ldiruvchisi va minori faqat ishora bilan farq qilishi mumkin. Buni sxematik ravishda $\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$ shaklda ko`rsatish mumkin.

Bu erda “+” ishora bilan shunday unsurlarning o`rni ko`rsatilganki, ularning algebraik to`ldiruvchi va minorlari tengdir, minus ishora bilan esa qolganlarining o`rni ko`rsatilgan.

Endi $\Delta = a_1A_1 + a_2A_2 + a_3A_3 = \begin{vmatrix} \boldsymbol{a_1} & \boldsymbol{b_1} & \boldsymbol{c_1} \\ \boldsymbol{a_2} & \boldsymbol{b_2} & \boldsymbol{c_2} \\ \boldsymbol{a_3} & \boldsymbol{b_3} & \boldsymbol{c_3} \end{vmatrix}$, $\begin{vmatrix} b_1 & b_1 & c_1 \\ b_2 & b_2 & c_2 \\ b_3 & b_3 & c_3 \end{vmatrix} = 0$ ekanligi ravshan

10-xossa.

Biror yo`l yoki ustun unsurlarini boshqa yo`l yoki ustun unsurlarining algebraik to`ldiruvchilari bilan ko`paytmalari yig`indisi nolga tengdir.

1.2.4. Yuqori tartibli aniqlovchilarni hisoblash

Bu erda bitta to`rtinchi tartibli aniqlovchini hisoblashni ko`rsatish bilan cheklanamiz. Hisoblashni nollar hosil qilish va ustun (yoki yo`l) unsurlari bo`yicha yoyish bilan ko`rsatamiz.

1) $\Delta=\begin{vmatrix}2&-1&3&5\\1&-2&4&6\\3&5&1&4\\-4&3&2&1\end{vmatrix}=\begin{vmatrix}2&-1&3&5\\-3&0&-2&-4\\13&0&16&29\\2&0&11&16\end{vmatrix}=\begin{vmatrix}-3&-2&-4\\13&16&29\\2&11&16\end{vmatrix}=-\frac{1}{9}\begin{vmatrix}3&6&12\\13&48&87\\2&33&48\end{vmatrix}=$

$=\frac{1}{9}\begin{vmatrix}3&0&0\\13&22&35\\2&29&40\end{vmatrix}=\frac{1}{3}\begin{vmatrix}1&0&0\\13&22&35\\2&29&40\end{vmatrix}=-\frac{1}{3}(22\cdot 40-35\cdot 29)=-\frac{5}{3}(176-203)=45.$

(2) $\Delta=2\cdot\begin{vmatrix}-2&4&6\\5&1&4\\3&2&1\end{vmatrix}-\begin{vmatrix}-1&3&5\\5&1&4\\3&2&1\end{vmatrix}+3\cdot\begin{vmatrix}-1&3&5\\-2&4&6\\5&2&1\end{vmatrix}+4\cdot\begin{vmatrix}-1&3&5\\-2&4&6\\5&1&4\end{vmatrix}=2\cdot\begin{vmatrix}-2&0&0\\5&11&19\\3&8&10\end{vmatrix}-$

$-\begin{vmatrix}-1&0&0\\5&16&29\\3&11&26\end{vmatrix}+3\cdot\begin{vmatrix}-1&0&0\\-2&-2&-4\\3&11&16\end{vmatrix}+4\cdot\begin{vmatrix}-1&0&0\\-2&-2&-4\\5&16&19\end{vmatrix}=-4\cdot(110-152)+(256-319)-$

$-3\cdot(-32+44)-4\cdot(-58+64)=168-123=45$

Yuqori tartibli aniqlovchi ikki usul bilan quyi tartibli aniqlovchilar bilan fodalanishi ma'lum:

a) 0 lar hosil qilish usuli;

b) biror yo`l yoki ustun elementlari, unsurlari bo`yicha yoyish usullari. Bu usullar nimalardan iborat.

Mavzu bo`yicha takrorlash savollari

1. Aniqlovchining nechta xossasini aytib bering?

2. Algebraik to`ldiruvchi nima?

3. Aniqlovchi minori nimava u qanday topiladi?

4. Aniqlovchi to`ldiruvchisi va minori qanday farq qiladi?

***1-* misol.** Qo`yidagi determinantlar hisoblansin.

$\begin{vmatrix}-5&3\\2&-1\end{vmatrix}$, $\begin{vmatrix}\cos 2x&-\sin 2x\\\sin 2x&\cos 2x\end{vmatrix}$, $\begin{vmatrix}tg3\alpha&1\\-1&ctg3\alpha\end{vmatrix}$,

Yechish (1) formulaga ko`ra hisoblaymiz:

1. $\Delta = \begin{vmatrix} -5 & 3 \\ 2 & -1 \end{vmatrix} = (-5)\cdot(-1) - 2\cdot 3 = 5\cdot 6 = -1.$

2. $\Delta = \begin{vmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{vmatrix} = \cos 2x \cdot \cos 2x + \sin 2x \cdot \sin 2x = \cos^2 2x + \sin^2 2x = 1.$

3. $\Delta = \begin{vmatrix} tg3\alpha & 1 \\ -1 & ctg3\alpha \end{vmatrix} = tg3\alpha \cdot ctg3\alpha - 1\cdot(-1) = 1+1 = 2.$

4. $\begin{vmatrix} 2 & 4 \\ 1 & 5 \end{vmatrix} = 10 - 4 = 6$; 5. $\begin{vmatrix} 4 & 3 & 6 \\ -3 & 2 & -5 \\ 1 & -1 & 7 \end{vmatrix} = 4\times 2\times 7 + 3\times(-5)\times 1 + 6\times(-3)\times$

$\times(-1) - 6\times 2\times 1 - 3\times(-3)\times 7 - 4\times(-5)\times(-1) = 56 - 15 + 18 - 12 + 63 - 20 = 90$

1.3-§ Chiziqli algebraik tenglamalar tizimini aniqlovchilar yordamida echish

Ushbu $\begin{cases} a_{11}x + a_{12}y = c_1, \\ a_2 x + a_{22}y = c_2 \end{cases}$ chiziqli sistemasi berilgan bo`lsin.

Sistemaning tenglamalaridan birinchisining har ikkala qismini a22 ga, ikkinchisini esaa12ga ko`paytirib, hosil bo`lgan tenglamalarini hadma-had qo`shib qo`yidagi natijaga ega bo`lamiz:

$$(a_{11}a_{12} - a_{12}a_{21})x = c_1 a_{22} - c_2 a_{12}$$

Shunga o`xshash, birinchi tenglamani $a_{21}$ ga, ikkinchisini esa $a_{11}$ga ko`paytirib, ularni hadma-had qo`shib,

$$(a_{11}a_{22} - a_{12}a_{21})y = c_2 a_{11} - c_1 a_{21}$$

tenglarni hosil qilamiz va tengliklarga ni tatbiq etamiz va qo`yidagi belgilashlarni kiritamiz:

$$\Delta = a_{11}a_{22} - a_{21}a_{12} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$\Delta_x = c_1 a_{22} - c_2 a_{12} = \begin{vmatrix} c_1 & a_{12} \\ c_2 & a_{22} \end{vmatrix}$$

$$\Delta_y = c_2 a_{11} - c_1 a_{21} = \begin{vmatrix} a_{11} & c_1 \\ a_{21} & c_2 \end{vmatrix}$$

$\Delta x, \Delta y$ - yordamchi determinantlar deyiladi.

Natijada sistemaga ekvivalent bo`lgan ushbu soda chiziqli tenglamalar sistemasini olishimiz mumkin: $\begin{cases} \Delta \cdot x = \Delta_x, \\ \Delta \cdot y = \Delta_y \end{cases}$

va tenglamalar sistemasi uchun qo`yidagi hollardan biri bo`lishi mumkin.

1. Agar $a_{11}a_{22} - a_{12}a_{21} \neq 0$ bo`lsa, u holda_ sistema yagona yechimga ega bo`lib, sistema echim qo`yidagicha topiladi:

$$x = \frac{c_1 a_{22} - c_2 a_{12}}{a_{11}a_{22} - a_{12}a_{21}}; \quad y = \frac{c_2 a_{11} - c_1 a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

yoki

$$x = \frac{\Delta_x}{\Delta} = \frac{\begin{vmatrix} c_1 & a_{12} \\ c_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & c_1 \\ a_{21} & c_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & b_{22} \end{vmatrix}}.$$

formulalar **Kramer** formulalari deyiladi.

2. Agar $\Delta = 0$ bo`lib $\Delta_x$ yoki $\Delta_y$ determinant yordamchi determinantlardan kamida bittasi noldan farqli bo`lsa,u holda _ sistema echimga ega emas yoki berilgan sistema tenglamalari birgalikda emas deyiladi.
3. Agar $\Delta = \Delta_x = \Delta_y = 0$ bo`lsa, sistemadagi birinchi tenglamaning koeffisiyentlari ikkinchi tenglamaning koeffisiyentlariga proporsional bo`ladi va sistema cheksiz ko`p echimga ega bo`ladi.

Kramer formulalarini quyidagicha ham hosil qilish mumkin. Faraz qilaylik ikki noma'lumli ikkita chiziqli algebraik tenglamalar tizimi berilgan bo`lsin:

(1) $\begin{cases} a_1 x + b_1 y = c_1, \\ a_2 x + b_2 y = c_2, \end{cases} \begin{vmatrix} b_2 \\ b_1 \end{vmatrix} \quad \begin{vmatrix} a_2 \\ a_1 \end{vmatrix}$ Sistema 1-tenglamasini b_2

ga 2- tenglamasini b_1 ga ko’paytirib hosil bo’lgan 1- tenglamadan 2-sini ayirib, so’ngra sistema 1-tenglamasini a_2 ga 2- tenglamasini a_1 ga ko’paytirib hosil bo’lgan 1- tenglamadan 2-sini ayirib, quyidagilarni hosil qilamiz.

$(a_1b_2-a_2b_1)\cdot x=c_1b_2-c_2b_1$, $(a_1b_2-a_2b_1)\cdot y=a_1c_2-a_2c_1$ Bulardan x va y no'ma'lumlarni topamiz.

$$x=\frac{c_1b_2-c_2b_1}{a_1b_2-a_2b_1}=\frac{\begin{vmatrix}c_1 & b_1\\ c_2 & b_2\end{vmatrix}}{\begin{vmatrix}a_1 & b_1\\ a_2 & b_2\end{vmatrix}} \qquad y=\frac{-c_1b_2+c_2b_1}{a_1b_2-a_2b_1}=\frac{\begin{vmatrix}a_1 & c_1\\ a_2 & c_2\end{vmatrix}}{\begin{vmatrix}a_1 & b_1\\ a_2 & b_2\end{vmatrix}} \qquad (2)$$

($a_1b_2-a_2b_1\neq 0$), bu erda

$\Delta=\begin{vmatrix}a_1 & b_1\\ a_2 & b_2\end{vmatrix}$ $\Delta_x=\begin{vmatrix}c_1 & b_1\\ c_2 & b_2\end{vmatrix}$ $\Delta_y=\begin{vmatrix}a_1 & c_1\\ a_2 & c_2\end{vmatrix}$ belgilashlarni kiritsak (2) ni,

$x=\frac{\Delta_x}{\Delta}$, $y=\frac{\Delta_y}{\Delta}$ (3) ko'rinishda yozish mumkin. Δ aniqlovchi (1) sistema-ning asosiy aniqlovchisi deyiladi.

Misol. Ushbu sistemani yeching

$$\begin{cases}3x+2y=12\\ x-2y=-4.\end{cases}$$

Quyidagi determinantlarni tuzamiz va ularni hisoblaymiz:

$$\Delta=\begin{vmatrix}3 & 2\\ 1 & -2\end{vmatrix}=-6-2=-8,$$

$$\Delta_x=\begin{vmatrix}12 & 2\\ -4 & 2\end{vmatrix}=-24+8=-16,$$

$$\Delta_y=\begin{vmatrix}3 & 12\\ 1 & -4\end{vmatrix}=-12-12=-24$$

$\Delta=-8\neq 0$ bo`lgani uchun sistema yagona yechimga ega. Kramer formulasiga ko`ra:

$$x=\frac{\Delta_x}{\Delta}=\frac{-16}{-8}=2,$$

$$y=\frac{\Delta_y}{\Delta}=\frac{-24}{-8}=3.$$

Demak to`g`ri chiziqlar xOy tekislikda (2;3) nuqtada kesishar ekan. Endi uch no'malumli uchna chiziqli ten uch no'malumli glamalar sistemasini Kramer usulida echishni ko'rib chiqaylik. Bizga quyidagi chiziqli tenglamalar tizimi – sistemasi berilgan bo`lsin.

$$\left.\begin{aligned} a_1x+b_1y+c_1z=d_1\\ a_2x+b_2y+c_2z=d_2\\ a_3x+b_3y+c_3z=d_3 \end{aligned}\right\} \quad (4)$$

Ikki noma`lumli ikkita chiziqli tenglamalar sistemasining umumiy yechimini topishda noma`lumlar oldidagi koeffitsiyentlarni tenglab birinchi tenglamadan ikkinchisini ayirish natijasi bitta noma`lumli tenglama hosil qilinib undan nom`lumning qiymati topilgan edi. Xuddi shu ishni sistemaga tadbiq etsak, natijada sistemaga ekvivalent quyidagi tenglamalar sistemasiga ega bo`ladi:

$$\begin{cases} x_1\cdot\Delta=\Delta_{x_1}\\ x_2\cdot\Delta=\Delta_{x_2}\\ x_3\cdot\Delta=\Delta_{x_3} \end{cases}$$

(4) ni algbraik qo'shish usulida echib Δ, $\Delta_{\tilde{o}}$, $\Delta_{\acute{o}}$, Δ_z aniqlovchilarni qiymatlarini topamiz (yuqoridagi kabi):

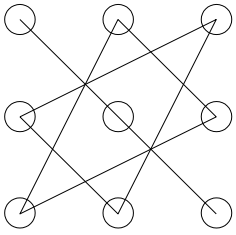
$$\Delta=\begin{vmatrix} a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3 \end{vmatrix},\quad \Delta_x=\begin{vmatrix} d_1 & b_1 & c_1\\ d_2 & b_2 & c_2\\ d_3 & b_3 & c_3 \end{vmatrix},\quad \Delta_y=\begin{vmatrix} a_1 & d_1 & c_1\\ a_2 & d_2 & c_2\\ a_3 & d_3 & c_3 \end{vmatrix},\quad \Delta_z=\begin{vmatrix} a_1 & b_1 & d_1\\ a_2 & b_2 & d_2\\ a_3 & b_3 & d_3 \end{vmatrix},$$

$$x=\frac{\Delta x}{\Delta},\quad y=\frac{\Delta y}{\Delta},\quad z=\frac{\Delta z}{\Delta},$$ (5) formulalar orqali (4) ning echimlarini (ildizlarini) topish mumkin. (3) va (5) formulalar Kramer formulalar deyiladi. (1) va (4) tenglamalar tizimlari echimga ega bo`lishi uchun $\Delta\neq 0$ bo`lishi zarur. Aniqlovchi (determinant)

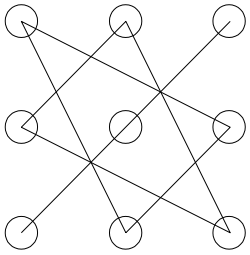
$$\Delta=a_1b_2c_3+a_2b_3c_1+a_3b_1c_2-a_3b_2c_1-a_2b_1c_3-a_1b_3c_2 \quad (6)$$

formula bilan hisoblanishini ko'zatish mumkin. Aniqlovchilar ba'zi masalalarni yechishda muhum bo`lgani uchun ularning xossalarini bilish zarurdir.

Bu xossalar har qanday tartibli aniqlovchilar uchun o`rinli bo`lsalar ham, bayon qilish oson bo'lishi uchun biz bu xossalarni faqat uchinchi tartibli aniqlovchilar uchun bayon qilamiz.

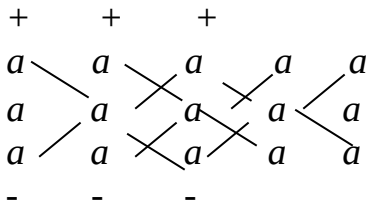


“+” ishora bilan



“-“ ishora bilan

Uchinchi tartibli determinantni Sarrius qoidasi (Elementlarni ko`chirish usuli ham deyiladi) deb ataluvchi usul bilan ham hisoblash mumkin. Bu usulni sxema ko`rinishida qoyidagicha yozish mumkin:



1- Misol. Qo`yidagi uchinchi tartibli determinantni hisoblaylik, uni yuqoridagi sxema ko`ra hisoblaymiz.

$$\begin{vmatrix} 2 & 1 & 3 \\ 1 & 0 & -1 \\ 4 & 5 & 2 \end{vmatrix}; \quad \begin{vmatrix} 2 & 1 & 3 \\ 1 & 0 & -1 \\ 4 & 5 & 2 \end{vmatrix} = (2\cdot 0\cdot 2 + 1\cdot(-1)\cdot 4 + 1\cdot 3\cdot 5) - (30\cdot 4 + 2\cdot 5 - (-1) + 1\cdot 1\cdot 2) =$$

$$= -4 + 15 - (-10 + 2) = 11 + 8 = 19$$

2-Misol. Ushbu sistemani Kramer usulida yechaylik

$$\begin{cases} x_1 + 2x_2 - x_3 = 2, \\ 2x_1 - 3x_2 + 2x_3 = 2, \\ 3x_1 + x_2 + x_3 = 8. \end{cases}$$

Yechish. Bu yerda

$$\Delta = \begin{vmatrix} 1 & 2 & -1 \\ 2 & -3 & 2 \\ 3 & 1 & 1 \end{vmatrix} = -8; \quad \Delta_{x_1} = \begin{vmatrix} 2 & 3 & -1 \\ 2 & -3 & 2 \\ 8 & 1 & 1 \end{vmatrix} = -8;$$

$$\Delta_{x_2} = \begin{vmatrix} 1 & 2 & -1 \\ 2 & 2 & 2 \\ 3 & 8 & 1 \end{vmatrix} = -16; \quad \Delta_{x_3} = \begin{vmatrix} 1 & 2 & 2 \\ 2 & -3 & 2 \\ 3 & 1 & 8 \end{vmatrix} = -24.$$

Ushbu $x=\frac{\Delta x}{\Delta}$, $y=\frac{\Delta y}{\Delta}$, $z=\frac{\Delta z}{\Delta}$, Kramer formulalarga asosan quyidagi echimlarni topamiz: $x_1=\frac{-8}{-8}=1$ $x_2=\frac{-16}{-8}=2$ $x_3=\frac{-24}{-8}=3$

3- misol. Ushbu $\begin{cases}2x_1+3x_2+x_3=2,\\ x_1-5x_2+2x_3=3,\\ 4x_1+6x_2+2x_3=0.\end{cases}$ tenglamalar sistemasi echimga ega emas, chunki $\Delta=0$

$\Delta=0$ va $\Delta_{x_1}=\Delta_{x_2}=\Delta_{x_3}=0$ bo`lsa, sistema yoki yechimga ega emas, yoki cheksiz ko`p echimga ega bo`ladi.

4-misol. Ushbu tenglamalar sistemasini yeching: $\begin{cases}x_1-x_2+2x_3=2,\\ 2x_1-2x_2+2x_3=4,\\ 3x_1-3x_2+6x_3=3.\end{cases}$

Yechish. Bu sistema uchun $\Delta=0,$ $\Delta_{x_1}=\Delta_{x_2}=\Delta_{x_3}=0$ Sistema yechimga ega emas.

5-misol. Ushbu tenglamalar sistemasini yeching $\begin{cases}x_1+2x_2+3x_3=0,\\ 2x_1+x_2-x_3=0,\\ 3x_1+3x_2+2x_3=0.\end{cases}$

Yechish. Sistema determinanti: $\Delta=\begin{vmatrix}1 & 2 & 3\\ 2 & 1 & -1\\ 3 & 3 & 2\end{vmatrix}=0;$

Sistemaning dastlabki ikkita tenglamasini olamiz:

$$\begin{cases}x_1+2x_2=-3x_3\\ 2x_1+x_2=x_3\end{cases}$$

Bu sistemani Kramer qoidasi bo`yicha yechamiz:

$$\Delta=\begin{vmatrix}1 & 2\\ 2 & 1\end{vmatrix}=1-4=-3;\quad \Delta_{x_1}=\begin{vmatrix}3x_3 & 2\\ x_3 & 1\end{vmatrix}=-5x_3;$$

$$\Delta_{x_2}=\begin{vmatrix}1 & 3x_3\\ 2 & x_3\end{vmatrix}=7x_3;$$

$$x_1 = \frac{\Delta_{x_1}}{\Delta} = \frac{-5x_3}{-3}; \quad x_2 = \frac{\Delta_{x_2}}{\Delta} = \frac{5}{3}x_3; \quad x_3 = \frac{\Delta_{x_3}}{\Delta} = \frac{7x_3}{-3} = -\frac{7}{3}x_3.$$

Demak, berilgan sistema cheksiz ko`p yechimga ega , chunki x_3 ga ixtiyoriy qiymatlar berib, x_1 va x_2 larning mos qiymatlarini topamiz. Masalan x_3=3 deb olsak: x_1 =5; x_2 =-7; x_3=9 deb olsak: x_2 =-21 larni topamiz va hokazolar.

1.4-§ Uchburchak yuzini hisoblash

Bir to`g`ri chiziqda yotmaydigan uchta $A(x_1,y_1)$, $B(x_2,y_2)$ va $C(x_3, y_3)$ nuqtalar berilgan bo`lsin. Faraz qilaylik AB=d, AC=d', φ esa ular orasidagi burchak bo`lsin. Ma'lumki $S_\Delta = \frac{1}{2}dd' \sin\varphi$ formula hosil bo`ladi. Bunda φ =±(θ'-θ) va θ' hamda θ lar esa mos ravishda AC va AB kesmalarning qutb burchaklaridir. Shuning uchun

$$S = \pm\frac{1}{2}dd' \sin(\theta' - \theta) = \pm\frac{1}{2}dd' (\sin\theta' \cos\theta - \cos\theta' \sin\theta) = \pm\frac{1}{2}(XY' - YX') =$$

$$= \pm\frac{1}{2}\cdot\begin{vmatrix} X & Y \\ X' & Y' \end{vmatrix} = \pm\frac{1}{2}\cdot\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix}.$$

Demak, uchburchak yuzini hisoblash uchun ushbu $\pm S = \frac{1}{2}\cdot\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix}$ formula hosil bo'ladi.

Mavzu bo`yicha takrorlash savollari

1. Aniqlovchi yordamida uchburchak yuzi qanday hisoblanadi. Bu formulani bilasizmi?

2. Qutb burchak deganda qanday burchakni tushinasiz?

II Bob
ANIQLOVCHILARNI KO’PBURCHAKLAR YUZALARINI HISOBLASHGA TADBIQI

Kirish

Kishilik jamiyatning dastlabki, paytlaridanoq turli shakldagi er maydonlari yuzalarini hisoblashga majbur bo‘lishgan va shu odamlar ish bilan shug‘ullanganlar.

Yuzalarni hisoblashda turli usullardan, ya'ni ma'lum bir xil shaklga ega yuzalarni hisoblashda formulalardan, ba'zi soha yuzasini hisoblashda yuzasi ma'lum bo‘lgan figura yuzasi bilan solishtirish usuli va hokozolardan foydalanilgan.

Avvalo eng sodda figuralar (uchburchak, to‘rtburchak, trapesiya, parrallelogramm, doira va h.zo.) yuzalarini hisoblashni o‘rganganlar, so‘ngra esa bu figuralarga qaraganda murakkab figuralar yuzalarini hisoblaganlar. Shunday qilib, kishilar turli shakldagi figuralar yuzalarini hisoblash usullarini o‘rganib va bu usullarni mukammallashtirib borganlar.

Matematika fani bir necha yuz asrlik taraqqiyoti davomida olamshumul muvaffaqiyatlarga erishdi. Buyuk olim aytganidek «Odamlar matematikaga uning boy xazinasiga qiziqqani uchun emas, balki uning yuz yillar davomida yaratilgan va sinalgan «apparatlari-usullari» bilan tanishishga va ulardan oqilona hamda unumli foydalanishga harakat qilganlar».

Ushbu uslubiy ishlanma ikki bo‘limdan iborat bo‘lib, birinchi bo‘limda ko‘pburchaklar yuzalarini uning uchlari koordinatalari orqali hisoblash va uning qo‘llanishlari, ikkinchi bo‘limda esa ko‘pburchak yuzasini kompleks sonlarni qo‘llab hisoblash va uning ba'zi bir tadbiqlari keltirilgan.

Ko‘pburchaklar yuzasini hisoblash haqida ba'zi mulohazalar

Bizga, maktab geometriya kursidan uchburchak, to‘rtburchak, trapesiya, paralellogramm va muntazam n-burchaklar kabi to‘g‘ri chiziq kesmalari bilan chegaralangan tekis shakllarni yuzalarini hisoblash formulalari ma'lum.

Ushbu bo‘limda, biz qavariq ko‘pburchak yuzasini uning uchlari koordinatlari orqali ifodalovchi formula, haqida so‘z yuritmoqchimiz.

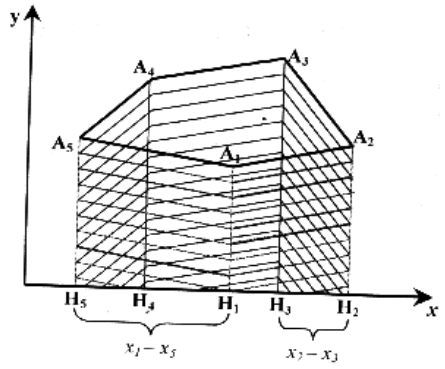
Bu formula orqali nafaqat uchburchakli, to‘rtburchakli hatto n-burchakli qavariq ko‘pburchak (muntazam bo‘lishi shart emas) yuzini ham hisoblash mumkin bo‘ladi. Ko‘pburchak yuzasini hisoblashda trapesiyalar va uchburchaklardan foydalanamiz. Bunday formula yuzalarni hisoblash uchun juda qulay hisoblanadi, chunki undan foydalanish uchun ko‘pburchak uchlarining koordinatalari ma'lum bo‘lsa kifoya, osonlikcha uning yuzasini hisoblash mumkin.

Masalaning qo‘yilishi:

Qavariq n-burchak yuzasini uning uchlari koordinatalari orqali ifodalovchi formulani toping. Bu masalani quyidagicha hal qilamiz.

2.1-§ Ko‘pburchak yuzini trapesiyalar yordamida hisoblash

Bizga, birinchi chorakda (x>0, y>0) joylashgan qavariq ko‘pburchak berilgan bo‘lsin. Uning uchlarini soat strelkasi yunalishiga qarama-qarshi (musbat) yo‘nalishda $A_1(x_1; y_1)$, $A_2(x_2; y_2)$,..., $A_n(x_n; y_n)$, lar orqali belgilaymiz. (1-chizma, n=5) Ko‘pburchakning har bir uchidan Ox o‘qiga A_1H_1, A_2H_2, ...,A_nH_n perpendikulyarlarni tushuramiz. Ularning uzunliklari mos ravishda y_1, y_2,...,y_n ga teng bo‘lsin**.**



1-чизма.

$A_kH_k\ H_{k+1}\ A_{k+1}$ trapesiyaning yuzi $\frac{(y_k + y_{k+1})(x_k - x_{k+1})}{2}$ (1) ifoda moduliga teng bo‘ladi, chunki uning qiymati $x_k>x_{k+1}$ da musbat, $x_k<x_{k+1}$ da esa manfiy bo‘ladi. (k=1…n; n+1 ni 1 bilan almashtiriladi, bizning holda). (1) o‘xshash n ta ifodalar yig‘indisi A_1A_2 ... A_n ko‘pburchak yuziga teng. Masalan, 1-chizmadagi

beshburchak uchun $\frac{(y_1+y_2)\cdot(x-x_2)}{2}$, $\frac{(y_2+y_3)\cdot(x_2-x_3)}{2}$, $\frac{(y_3+y_4)\cdot(x_3-x_4)}{2}$, $\frac{(y_4+y_5)\cdot(x_4-x_5)}{2}$, $\frac{(y_5+y_1)\cdot(x_5-x_1)}{2}$ beshta ifodadan yuqori tomoniga mos keluvchi uchtasi musbat, quyi tomoniga mos keluvchi ikkitasi manfiy («pastki»); «yuqori» trapesiyalar yuzalari yig‘indisidan «pastki» trapesiyalar yuzalari yig‘indisini «ayirib» beshburchak yuzasini topamiz (1-chizma).

Яъни $S_5=\frac{(y_1+y_2)\cdot(x_1-x_2)}{2}+\frac{(y_2+y_3)\cdot(x_2-x_3)}{2}+\frac{(y_3+y_4)\cdot(x_3-x_4)}{2}+$

$+\frac{(y_4+y_5)\cdot(x_4-x_5)}{2}+\frac{(y_5+y_1)\cdot(x_5-x_1)}{2}=\frac{x_1y_2-x_2y_1}{2}+\frac{x_2y_3-y_2x_3}{2}+$

$+\frac{x_3y_4-y_3x_4}{2}+\frac{x_4y_5-x_5y_4}{2}+\frac{y_1x_5-x_1y_5}{2}=\frac{1}{2}[(x_1y_2-x_2y_1)+(x_2y_3-x_3y_2)+]$

$+(x_3y_4-x_4y_3)+(x_4y_5-x_5y_4)+(x_5y_1-x_1y_5)]=\frac{1}{2}\left[\begin{vmatrix}x_1 & x_2\\ y_1 & y_2\end{vmatrix}+\begin{vmatrix}x_2 & x_3\\ y_2 & y_3\end{vmatrix}+\begin{vmatrix}x_3 & x_4\\ y_3 & y_4\end{vmatrix}+\begin{vmatrix}x_4 & x_5\\ y_4 & y_5\end{vmatrix}+\begin{vmatrix}x_5 & x_1\\ y_5 & y_1\end{vmatrix}\right]$ (2)

Izoh 1. $a_1b_2-b_1a_2$ songa 2-tartibli aniqlovchi deyiladi va $\begin{vmatrix}a_1 & a_2\\ b_1 & b_2\end{vmatrix}$ kabi belgilanadi.

Ixtiyoriy n burchakli qavariq ko‘pburchak yuzasi uchun ham (2) formulaga o‘xshash formulani keltirib chiqarish mumkin va u quyidagi ko‘rinishda bo‘ladi.

$$S_n=\frac{1}{2}[(x_1y_2-x_2y_1)+(x_2y_3-x_3y_2)+\ldots+(x_ny_1-x_1y_n)] \quad (3)$$

(3) formulani shartli ravishda $S_n=\frac{1}{2}\left[\begin{vmatrix}x_1x_2x_3x_4\ldots x_{n-1}x_nx_1\\ y_1y_2y_3y_4\ldots y_{n-1}y_ny_1\end{vmatrix}\right]$ (4)

ko‘rinishda yozish ham mumkin, yoki 2- tartibli aniqlovchilardan foydalansak (4) ni yanada ixcham va tushinarli qilib

$$S_n=\frac{\begin{vmatrix}x_1 & x_2\\ y_1 & y_2\end{vmatrix}+\begin{vmatrix}x_2 & x_3\\ y_2 & y_3\end{vmatrix}+\begin{vmatrix}x_3 & x_4\\ y_3 & y_4\end{vmatrix}+\ldots+\begin{vmatrix}x_4 & x_1\\ y_n & y_1\end{vmatrix}}{2}$$ (5) ko’rinishda yozamiz.

Agar, ko‘pburchak uchlarini soat strelkasi yo‘nalishi (manfiy) bo‘yicha belgilasak yoki koordinata o‘qlarini belgilanishini almashtirsak Oy→Ox u holda

ko‘pburchakni $a_1, a_2, a_3, ..., a_n, a_1$ tomonlariga mos keluvchi (3) formuladagi har bir kichik qavs ichidagi ayirma ishorasi teskarisiga o‘zgaradi.

Demak, agar ko‘pburchakda yo‘nalish «musbat» bo‘lsa, uning yuzasi musbat ishorali «manfiy» bo‘lsa manfiy ishorali bo‘lar ekan, shuning uchun (3) formulada kvadrat qavs o‘rniga modul ishorasini quysak uning qiymati doimo musbat bo‘ladi,

ya'ni $$S_n = \frac{1}{2}\left|\sum_{k=1}^{n}\left(x_k y_{k+1} - x_{k+1} y_k\right)\right| \qquad (6)$$

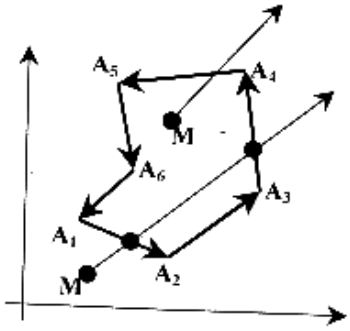
Bu erda x_{k+1}, x_1 bilan; y_{k+1} esa y_1 bilan almashtiriladi.

Biz, yuqoridagi (5) formulani keltirib chiqarishda, ko‘pburchak uchlari koordinatalarini musbat, uning yuzini esa qavariq deb faraz qilgan edik. Lekin (6) formula ko‘pburchak uchlarining koordinatalarining ixtiyoriy ishorali va uning o’zi qavariq bo‘lmasa ham o‘rinli bo‘lishini isbotlash mumkin.

Koordinatalari turli ishorali, har qanday $A_1, A_2, A_3, .. , A_n$ ko‘pburchakni biror $\vec{a} = \{a_x; a_y\}$ vektorga parallel ko‘chirish yordamida 1-chorakka o‘tkazish mumkin, u holda uning uchlarining koordinatalari musbat ishorali ko‘pburchak uchun (5) formula o‘rinli, demak u berilgan dastlabki ko‘pburchak uchun ham o‘rinli.

(5) formulani ixtiyoriy qavariq bo‘lmagan ko‘pburchak uchun o‘rinli bo‘lishini quyidagi lemmaga asosan isbotlash mumkin.

Lemma. Faraz qilaylik MP – biror bir M nuqtadan chiquvchi nur bo‘lsin. Agar M nuqta ko‘pburchak tashqarisida yotsa, u holda uni “musbat” yo‘nalish bo‘yicha aylanganda, MP nurni juft marta kesib o‘tamiz (o‘ngdan chapga va chapdan o‘ngga teng sonda). Agar M nuqta kœpburchak ichida yotsa tog‘ marta (œngdan chapga bir marta ortiq).

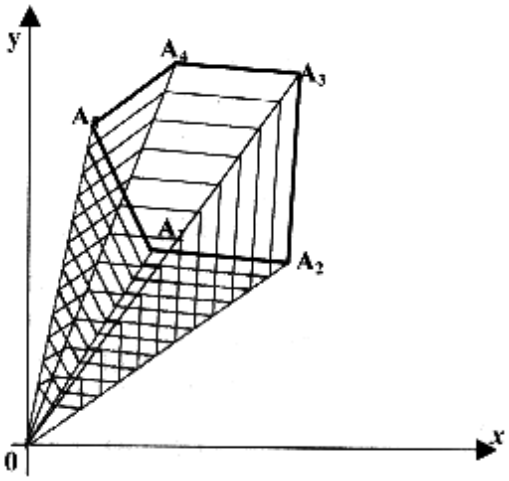


Izoh 2. Lemmani parallel ko‘chirishning mavjudligi va yagonaligi teoremasiga ko‘ra isbotlash mumkin

2-chizma

2.2-§ Ko‘pburchak yuzini uchburchaklar yuzalari yordamida hisoblash

Yuqorida keltirilgan (5) asosiy formulani uchburchaklar yuzasini hisoblash formulalaridan ham foydalanib keltirib chiqarish mumkin va unda har bir $\frac{X_к Y_{к+1} - X_{к+1} Y_к}{2}$ qo‘shiluvchining geometrik ma'nosi aniq bo‘ladi. $(X_к \succ 0; Y_к \succ 0)$.



3-chizma

Ushbu yig‘indiga faqat ikkita qo‘shni $A_к$ *va* $A_{к+1}$ nuqtalarning koordinatalari kiradi. $\frac{X_к Y_{к+1} - X_{к+1} Y_к}{2}$ ifodaning qiymati $OA_к A_{к+1}$ uchburchakning yuziga teng, hamda agar $OA_{к+1}$ kesma $OA_к$ kesmaga qaraganda Ox yoki musbat yo‘nalishi bilan katta burchak tashkil etsa, uning qiymati «musbat» aks holda «manfiy» bo’ladi (3-chizma). undan, $\frac{X_к Y_{к+1} - X_{к+1} Y_к}{2}$ ga o’zshash **n** та ifoda yig‘indisi n-burchakli ko‘pburchak yuzasiga tengligi kelib chiqadi. Uchinchi chizmaga, ko‘ra «yuqori» uchburchaklar yuzasidan, «pastki» uchburchaklar yuzasini ayirib kupburchak yuzasini xosil kilish mumkin ekanligi kelib chiqadi.

(5) formulani matematik induksiya metodi yordamida ham isbotlash mumkin.

Buning uchun qaralayotgan masalani bir necha bo‘lakka bo‘lib qaraymiz, ya'ni:

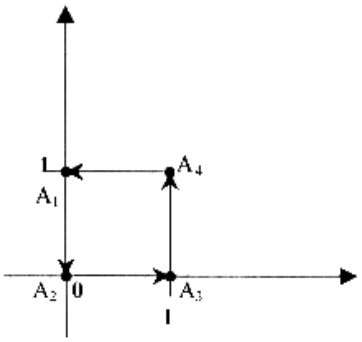
1) Avvalo (5) formulani to‘g‘ri burchakli, katetlari koordinata o‘qlariga parallel bo‘lgan, so‘ngra biror tomoni koordinata o‘qlaridan biriga parallel bœlgan keyin esa ixtiyoriy uchburchak uchun isbotlaymiz.

2) Ixtiyoriy qavariq to‘rtburchak uchun isbotlaymiz, buning uchun to‘rtburchakni diogonal yordamida ikkita uchburchakka ajratamiz.

3) Qaralayotgan qavariq n-burchakni, $A_2 A_3 .... A_n$ $(n-1)$- burchak va $A_1 A_2 A_3$ uchburchak birlashmasidan tashkil topgan deb, matematik induksiya metodini qo‘llaymiz.

2.2.1. Yuqorida o’rganilgan mavzularni masalalar yechishga qo‘llanilishi

Endi yuqoridagi formulani qo‘llab quyidagi masalalarni echaylik.



4-chizma

1- masala. Uchlari (0;0), (0;1), (1;0), (1;1) nuqtalarda bo‘lgan to‘rtburchak yuzini (5) formula yordamida hisoblaylik.(4-chizma)

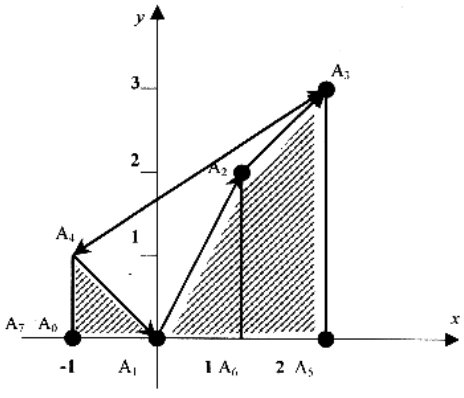
Echish:

a) musbat yo‘nalish bo‘yicha

$$S=\frac{1}{2}\begin{vmatrix}0 & 0\\ 1 & 0\end{vmatrix}+\begin{vmatrix}0 & 1\\ 0 & 0\end{vmatrix}+\begin{vmatrix}1 & 1\\ 0 & 1\end{vmatrix}+\begin{vmatrix}1 & 0\\ 1 & 1\end{vmatrix}=2\cdot\frac{1}{2}=1.$$

b) manfiy yo‘nalish bo‘yicha

$$S=\frac{1}{2}\left\|\begin{matrix}0 & 1\\ 1 & 1\end{matrix}\right|+\begin{vmatrix}1 & 1\\ 1 & 0\end{vmatrix}+\begin{vmatrix}1 & 0\\ 0 & 0\end{vmatrix}+\left\|\begin{matrix}0 & 0\\ 0 & 1\end{matrix}\right\|=\frac{1}{2}|0-1-1+0+0|=1.$$



5-chizma

Biz uchlari berilgan nuqtalarda yotuvchi to‘rtburchak yuzasini ikki xil yo‘nalishda xisoblab bir xil natijaga keldik.

v) berilgan figura, tomonining uzunligi 1 ga bo‘lgan teng kvadrat bo‘lgani uchun uning yuzi $S=1^2=1$ ga teng bo’ladi..

Xulosa: Qaralayotgan figura yuzasi (5) formula yordamida to‘g‘ri hisoblangan.

2- **masala**. Uchlari (0;0), (2;3), (1;2), (-1;1) nuqtalarda joylashgan qavariq bo‘lmagan to‘rtburchak yuzini hisoblang. (5-chizma).

Echish.

a) manfiy yo‘nalish bo‘yicha

$$S=\frac{1}{2}\left\|\begin{matrix}0 & -1\\ 0 & 1\end{matrix}\right|+\left|\begin{matrix}-1 & 2\\ 1 & 3\end{matrix}\right|+\left|\begin{matrix}2 & 1\\ 3 & 2\end{matrix}\right|+\left\|\begin{matrix}1 & 0\\ 2 & 0\end{matrix}\right|=\frac{1}{2}|-3-2+4-3|=\frac{1}{2}|-4|=2$$

б) musbat yo‘nalish bo‘yicha

$$S=\frac{1}{2}\left\|\begin{matrix}0 & 1\\ 0 & 2\end{matrix}\right|+\left|\begin{matrix}1 & 2\\ 2 & 3\end{matrix}\right|+\left|\begin{matrix}2 & -1\\ 3 & 1\end{matrix}\right|+\left\|\begin{matrix}-1 & 0\\ 1 & 0\end{matrix}\right|=\frac{1}{2}|-1+5|=\frac{1}{2}\cdot 4=2.$$

v) $A_1A_2A_3A_4$ qavariq bo‘lmagan to‘rtburchakdir. Xaqiqatdan xam a) va b) bandlarda olingan natijani to‘g‘riligini quyidagicha isbotlash mumkin. Buning uchun $A_0A_5A_3A_4$ trapesiya yuzasidan $A_1A_4A_0, A_1A_6A_2$ uchburchaklar va $A_6A_5A_3A_2$ trapesiya yuzalari yig‘indisini ayirib masala to‘g‘ri echilganini ko‘rish mumkin.

Haqiqatan ham, $S=\frac{1+3}{2}\cdot 3-\frac{1\cdot 2}{2}-\frac{2+3}{2}\cdot 1-\frac{1\cdot 1}{2}=6-1-2{,}5-0{,}5=2$

3-Masala.

Qanday shart bajarilganda $(x_1;y_1)$; $(x_2;y_2)$; $(x_3;y_3)$ kordinatali nuqtalar bir to‘g‘ri chiziqda yotadi.

Echish: Agar $A_1(x_1;y_1)$; $A_2(x_2;y_2)$; $A_3(x_3;y_3)$ nuqtalar bitta to‘g‘ri chiziqda yotsa, ularga qurilgan uchburchakning yuzi 0 ga teng bo‘ladi. (5) formuladan n=3 da kuyidagi shartni hosil qilamiz.

$$S=\frac{1}{2}|(x_1y_2-y_1x_2)+(x_2y_3-x_3y_2)+(x_3y_1-x_3y_1)|=0$$

Endi yuqoridagi formuladan foydalanib quyidagi teoremalarni isbotlaylik.

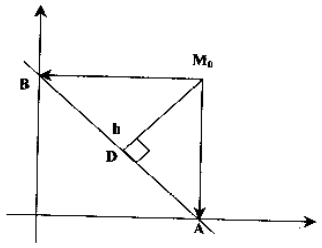
2,2,2. Ba'zi bir teoremalar

1-Teorema. Agar A va B nuqtalar $ax+by+c=0$ to‘g‘ri chiziqning koordinata o‘qlari bilan kesishish nuqtalari bo‘lsa, u holda $M_0(x_0:y_0)$ nuqtadan $ax+by+c=0$ to’g’ri chiziqgacha bo’lgan masofa $d=\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$ ga teng bo‘ladi.

Isbot: To‘g‘ri chiziq ax + by + c =0 tenglama bilan berilgan va $M_o(x_o;y_o)$ esa to‘g‘ri chiziqqa tegishli emas, berilgan to‘g‘ri chiziq Ox o‘qi bilan kesishsa

y =0 bo‘ladi va $A\left(-\frac{c}{a};0\right)$ huqta Oy o’qi bilan kesishsa $x=0; y=-\frac{c}{b}$ demak,

$B\left(0;-\frac{c}{b}\right)$ nuqta hosil bo‘ladi. (6-chizma)



6-chizma

Endi uchlari A, B va M_0 nuqtalarda yotuvchi uchburchak yuzini (3) formulaga asosan topamiz.

$$S=\frac{1}{2}\left|\begin{vmatrix}\frac{-c}{a} & x_o\\ o & y_o\end{vmatrix}+\begin{vmatrix}x_o & o\\ y_o & \frac{-c}{b}\end{vmatrix}+\begin{vmatrix}o & \frac{-c}{a}\\ \frac{-c}{b} & o\end{vmatrix}\right|=\frac{1}{2}\left|-\frac{cy_o}{a}-\frac{cx_o}{b}-\frac{c^2}{ab}\right|=\frac{1}{2}\frac{c|ax_o+by_o+c|}{ab}$$

M_oD uchburchak balandligi va M_o nuqtada $ax+by+c=0$ to‘g‘ri chiziqgacha bo‘lgan masofadir.

$$S=\frac{1}{2}\cdot|AB|\cdot|M_oD|$$

$$|AB|=\sqrt{\frac{c^2}{a^2}+\frac{c^2}{b^2}}=\frac{c}{ab}\sqrt{a^2+b^2}$$

Demak: $\frac{1}{2}\frac{c}{ab}\cdot|ax_0+by_0+c|=\frac{1}{2}\frac{c}{ab}\cdot\sqrt{a^2+b^2}\cdot d$ bundan esa, $d=\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$

ekanligini hosil qilamiz.

Izox 3. Ushbu formula «Analitik geometriya» kursida boshqa usul bilan isbotlanadi.

2-Teorema. k – koffisientli gomotetiyada ko‘pburchak yuzi k^2 marta ortadi.

Isbot. Ko‘pburchak yuzini hisoblash formulasiga asosan, k – koeffisentli gomotetiyada uning yuzasi k^2 marta ortishini ko‘rsataylik.

$$S=\frac{1}{2}\left|\sum_{k=1}^{n}(x_{k+1}y_k-y_{k+1}x_k)\right|=\begin{bmatrix}x_i=kx_i\\ y_i=ky_i\end{bmatrix}=\frac{1}{2}\left|\sum_{k=1}^{n}k\cdot x_{k+!}\cdot k\cdot y_k-k\cdot y_{k+1}\cdot k\cdot x_k\right|=$$

$$=\frac{1}{2}k^2\cdot\left|\sum_{k=1}^{n}(x_{k+1}y_k-y_{k+1}\cdot x_k)\right|=k^2S$$

3-Teorema. $x=\kappa x,\quad y=y/\kappa$ (k – tayin son) giperbolik almashtirishda, ko‘pburchak yuzi o‘zgarmaydi.

Isbot. Bu teoremani isbotlash uchun (5) formuladan foydalanamiz.

$$S=\frac{1}{2}\left|\sum_{i=1}^{n}(x_iy_{i+1}-x_{i+1}y_i)\right|=\begin{bmatrix}x^1=kx & \ddot{e}\kappa u & x=x^1/k\\ y^1=y/k & & y=y^1k\end{bmatrix}=\frac{1}{2}\left|\sum_{i=1}^{n}(\frac{x_i^1}{k}\cdot ky_{i+1}^1-\frac{x_{i+1}^1}{k}\cdot ky_i^1)\right|=$$

$$=\frac{1}{2}\left|\sum_{i=1}^{n}(x_i^1\cdot y_{i+1}^1-x_{i+1}^1\cdot y_i^1)\right|$$

Mustaqil yecgish uchun misollar:

***Hisoblang*: 1.** $\begin{vmatrix}3&-4\\4&6\end{vmatrix}$ **2.** $\begin{vmatrix}3&2\\5&-4\end{vmatrix}$ **3.** $\begin{vmatrix}\sqrt{b}&-1\\b&\sqrt{b}\end{vmatrix}$ **4.** $\begin{vmatrix}\cos\alpha&-\sin\alpha\\\sin\alpha&\cos\alpha\end{vmatrix}$

5. $\begin{vmatrix}35&-6\\4&0\end{vmatrix}$ **6.** $\begin{vmatrix}7&0\\5&-2\end{vmatrix}$

Aniqlovchilarni uchburchak usulida hisoblang:

7. $\begin{vmatrix}2&3&4\\5&-2&1\\-3&2&2\end{vmatrix}$ **8.** $\begin{vmatrix}-4&2&3\\1&1&4\\-3&2&2\end{vmatrix}$ **9.** $\begin{vmatrix}4&-3&2\\1&2&3\\2&-2&3\end{vmatrix}$ **10.** $\begin{vmatrix}-2&1&2\\3&2&4\\3&2&5\end{vmatrix}$

11. $\begin{vmatrix}12&6&-4\\6&4&4\\3&2&8\end{vmatrix}$ **12.** $\begin{vmatrix}3&7&2\\6&0&4\\3&2&8\end{vmatrix}$ **13.** $\begin{vmatrix}1&1&-1\\1&-1&1\\-1&1&1\end{vmatrix}$ **14.** $\begin{vmatrix}1&17&-7\\-1&13&1\\-1&7&1\end{vmatrix}$

15. $\begin{vmatrix}2&0&5\\1&3&16\\0&-1&10\end{vmatrix}$ **16.** $\begin{vmatrix}1&2&4\\-2&1&-3\\3&-4&2\end{vmatrix}$ **17.** $\begin{vmatrix}2&19&30\\0&-5&-12\\0&2&5\end{vmatrix}$ **18.** $\begin{vmatrix}4&3&5\\6&7&1\\9&1&8\end{vmatrix}$

19. $\begin{vmatrix}0&1&0\\-4&4&0\\-2&1&2\end{vmatrix}$ **20.** $\begin{vmatrix}-1&3&-2\\-4&1&2\\3&-4&4\end{vmatrix}$ **21.** $\begin{vmatrix}1&-3&3\\-2&-6&13\\-1&-4&8\end{vmatrix}$ **22.** $\begin{vmatrix}1&-1&-1\\-1&4&7\\8&1&-1\end{vmatrix}$

23. $\begin{vmatrix} 4 & -5 & 7 \\ 1 & -4 & 9 \\ -4 & 0 & 5 \end{vmatrix}$ **24.** $\begin{vmatrix} 9 & 3 & 5 \\ 2 & 0 & 3 \\ 0 & 1 & -1 \end{vmatrix}$ **25.** $\begin{vmatrix} 4 & -5 & 2 \\ 5 & -7 & 3 \\ 6 & -9 & 4 \end{vmatrix}$ **26.** $\begin{vmatrix} -3 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & -1 & 3 \end{vmatrix}$

Tenglamadan x ni toping.

27. $\begin{vmatrix} x^2 & 4 & 9 \\ x & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 0$ **28.** $\begin{vmatrix} x^2 & 3 & 2 \\ x & -1 & 1 \\ 0 & 1 & 4 \end{vmatrix} = 0$ **29.** $\begin{vmatrix} 1 & 3 & x \\ 4 & 5 & -1 \\ 2 & -1 & 5 \end{vmatrix} = 0$

Hisoblang:

30. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 1 \\ 1 & 2 & 2 & 1 \\ 3 & 1 & 2 & 1 \end{vmatrix}$ **31.** $\begin{vmatrix} 2 & -1 & 3 & 4 \\ 0 & -1 & 5 & -3 \\ 0 & 0 & 5 & -3 \\ 0 & 0 & 0 & 2 \end{vmatrix}$ **32.** $\begin{vmatrix} 2 & -1 & 1 & 0 \\ 0 & 1 & 2 & -1 \\ 3 & -1 & 2 & 3 \\ 3 & 1 & 6 & 1 \end{vmatrix}$

33. $\begin{vmatrix} 2 & 3 & -3 & 4 \\ 2 & 1 & -1 & 2 \\ 6 & 2 & 1 & 0 \\ 2 & 3 & 0 & -5 \end{vmatrix}$

Tengsizlikni eching.

$\begin{vmatrix} x^2 & 4 & 9 \\ x & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 0$ **34.** $\begin{vmatrix} 3 & -2 & 1 \\ 1 & x & -2 \\ -1 & 2 & -1 \end{vmatrix} < 1$ **35.** $\begin{vmatrix} 2 & x+2 & -1 \\ 1 & 1 & -2 \\ 5 & -3 & x \end{vmatrix} > 0$ 36. $\begin{vmatrix} 4 & -5 & 2 \\ 5 & -7 & 3 \\ 6 & -9 & 4 \end{vmatrix} < 0$ 37.

Mavzu bo`yicha takrorlash savollari

1. Aniqlovchi (determinant) nima?
2. Determinant tartibi qanday aniqlanadi.?
3. Kramer formulalarini yozing?
4. Uchinchi tartibli aniqlovchi qanday hisoblanadi?

III Bob MATRITSA TUSHUNCHASI

3.1-§ Matritsalar haqida ba'zi bir ta'riflar. Matritsalarni qo`shish va songa ko`paytirish. Matritsalarni ko`paytirish. Chiziqli tenglamlar tizimini matritsa shaklida yozish va matritsa usulida echish

Ta'rif 1. mxn ta elementdan iborat bo'lib m yo`l va n ustundan tashkil topgan

$$A=\begin{Vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{Vmatrix} \text{ yoki } A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}=\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$ qisqacha

$A=\|a_{ij}\|$ bunda $i=\overline{1,m}$ $j=\overline{1,n}.$ jadvalga matritsa deyiladi, bu erda a_{ij} - matritsa hadlari deyiladi.

Agar m=n bo`lsa matritsa kvadrat matritsa deyiladi.

Ta'rif 2. $\Delta(A)=\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$ aniqlovchi kvadrat matritsa aniqlovchisi deyiladi.

Izoh. Nokvadrat matritsa aniqlovchiga ega emas.

Ta'rif 3. Agar A matritsa yo`llari $A^*$ matritsa ustunlari bo`lsa, u vaqtda A^* matritsa A matritsaga nisbatan ***transponirlangan*** matritsa deyiladi.

Misol: $A=\begin{Vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{Vmatrix}$ bo'lsa $A^*=\begin{Vmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{Vmatrix}$ bo'ladi

Ta'rif 4. $\begin{Vmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ 0 & 0 & \dots & a_{23} \end{Vmatrix}$ diagonal matritsa deyiladi.

Ta'rif 5. $\begin{Vmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ 0 & 0 & \dots & 1 \end{Vmatrix}$ kvadrat matritsa, birlik matritsa deyiladi

Ta'rif 6. $\begin{Vmatrix} x_1 \\ x_2 \\ . \\ x_n \end{Vmatrix}$ ustun matritsa va $\| x_1 \quad x_2 \ldots x_n \|$ yo`l matritsa deyiladi.

Ta'rif 7. Agar matritsalarning yo`llari va ustunlari sonlari mos ravishda teng bo`lib bu matritsalarning mos unsurlari-elementlari ham teng bo`lsa, bunday matritsalar teng deyiladi.

Ta'rif 8. $B=\begin{pmatrix} b_{11} & b_{12} & b_{13} & \ldots & b_{1q} \\ 0 & b_{22} & b_{23} & \ldots & b_{2q} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & b_{pq} \end{pmatrix}$ uchburchakli matrisa

Mavzu bo`yicha takrorlash savollari

1. Matritsa nima? Qanday matritsa uchun aniqlovchi mavjud bo`ladi?
2. Transponirlangan matritsa qanday matritsa?
3. Dioganal matritsa va birlik matritsalar qanday yoziladi?
4. Ustun va yo`l matritsalar qanday matritsalar?
5. Qanday matritsalar teng matritsalar deyiladi?

3.1.2 Teskari almashtirish va uni topish

Faraz qilaylik matritsa $A=\begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix}$ bo'lgan (1) $\begin{cases} y_1 = a_{11}x_1 + a_{12}x_2 \\ y_2 = a_{21}x_1 + a_{22}x_2 \end{cases}$

chiziqli almashtirish berilgan bo`lsin. (1) chiziqli almashtirish Ox_1x_2 tekislikning bitta nuqtasiga Oy_1y_2 tekislikning bitta nuqtasini mos qo'yadi.

Agar $\Delta(A)=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$ bo`lsa, u vaqtda (1) yagona $x_1 = \dfrac{\begin{vmatrix} y_1 & a_{12} \\ y_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$, $x_2 = \dfrac{\begin{vmatrix} a_{11} & y_1 \\ a_{21} & y_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$

yechimga ega bo'ladi. Bu yerda $\left.\begin{aligned} x_1 &= \frac{a_{22}}{\Delta}\cdot y_1 + \frac{-a_{12}}{\Delta}\cdot y_2 \\ x_2 &= \frac{-a_{21}}{\Delta}\cdot y_1 + \frac{a_{11}}{\Delta}\cdot y_2 \end{aligned}\right\}$ (3)

chiziqli almashtirish (1) ga nisbatan teskari chiziqli almashtirish deyiladi. Bu holda (1) almashtirish o`zaro birma bir akslantirish deyiladi, chunki bu holda Oy_1y_2

tekislikning har bir $M(y_1,y_2)$ nuqtasiga Ox_1x_2 tekislikning bitta $M(x_1,x_2)$ nuqtasi mos keladi. (3) almashtirish matritsasini A^{-1} bilan belgilaymiz: $A^{-1}=\begin{Vmatrix} \frac{a_{22}}{\Delta} & \frac{-a_{12}}{\Delta} \\ \frac{-a_{21}}{\Delta} & \frac{a_{11}}{\Delta} \end{Vmatrix}$

Mavzu bo`yicha takrorlash savollari

1. Berilgan chiziqli almashtirish va bu almashtirishga teskari almashtirish qanday ko'rinishda bo'ladi?
2. Teskari almashtirish qanday shartda mavjud bo'ladi?

3.2-§ Matritsalar ustida amallar

3.2.1. Matrisalarni qo`shish.

Matrisalar algebraik yig'indisini topish uchun ular bir xil tartibli bo'lishlari shart, aks holda ularni algebraik yig'indisini topishmumkin emas. Agar bir hil o`lchamli ikkinchi tartibli kvadrat matrisalar berilgan bo`lsa,

$A=\begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix}$, va $B=\begin{Vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{Vmatrix}$, u holda ularning algbraik yig`indisi deb

$C=\begin{Vmatrix} a_{11}\pm b_{11} & a_{21}\pm b_{12} \\ a_{21}\pm b_{21} & a_{22}\pm b_{22} \end{Vmatrix}$ matrisaga aytiladi.

1-misol. $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}+\begin{bmatrix} -4 & 5 \\ 2 & 1 \end{bmatrix}=\begin{bmatrix} 3-4\cdot 1+5 \\ 0+2\cdot 2+1 \end{bmatrix}=\begin{bmatrix} -1 & 6 \\ 2 & 3 \end{bmatrix}.$

2-misol. Bu misolda ikkita bir xil tartibli to'trburchakli matrisalar yig'indisini topaylik/

$$\begin{bmatrix} 1 & 3 \\ 2 & 1 \\ -1 & 4 \end{bmatrix}+\begin{bmatrix} 2 & -1 \\ 0 & 3 \\ 4 & -2 \end{bmatrix}=\begin{bmatrix} 1+2 & 3-1 \\ 2+0 & 1+3 \\ -1+4 & 4-2 \end{bmatrix}=\begin{bmatrix} 3 & 2 \\ 2 & 4 \\ 3 & 2 \end{bmatrix}.$$

3- Misol: $A=\begin{pmatrix} 2 & 4 & 1 \\ -1 & 0 & 2 \end{pmatrix}$ $B=\begin{pmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ A+B=? A-B=? 2A-3B=?

A+B=$\begin{pmatrix} 2+0 & 4+2 & 1+1 \\ -1+1 & 0+1 & 2+2 \end{pmatrix}=\begin{pmatrix} 2 & 6 & 2 \\ 0 & 1 & 4 \end{pmatrix}$

A-B=$\begin{pmatrix} 2-0 & 4-2 & 1-1 \\ -1-1 & 0-1 & 2-2 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 0 \\ -2 & -1 & 0 \end{pmatrix}$

2A-3B = $2\begin{pmatrix} 2 & 4 & 1 \\ -1 & 0 & 2 \end{pmatrix} - 3\begin{pmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 8 & 2 \\ -2 & 0 & 4 \end{pmatrix} - \begin{pmatrix} 0 & 6 & 3 \\ 3 & 3 & 6 \end{pmatrix} =$

==$\begin{pmatrix} 4 & 2 & -1 \\ -5 & -3 & -2 \end{pmatrix}$

3.2.2. Matrisani songa ko`paytirish

$A = \begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix}$, matrisaning $\lambda \neq 0$ songa ko`paytmasi deb, $\lambda \cdot A = \begin{Vmatrix} \lambda \cdot a_{11} & \lambda \cdot a_{12} \\ \lambda \cdot a_{21} & \lambda \cdot a_{22} \end{Vmatrix}$ matrisaga aytiladi. Agar $\lambda = 0$ bo'lsa, quyidagi tenglik o'rinli bo'ladi $\begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix} \cdot 0 = \begin{Vmatrix} 0 & 0 \\ O & O \end{Vmatrix}$.

3.2.3. Matrisalarni ko`paytirish

Agar $A = \begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix}$ va $B = \begin{Vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{Vmatrix}$ matritsalar berilgan bo`lsa, u vaqtda B va A matritsalar ko`paytmasi

$B \cdot A = \begin{Vmatrix} b_{11}a_{11} + b_{12}a_{21} & b_{11}a_{12} + b_{12}a_{22} \\ b_{21}a_{11} + b_{22}a_{21} & b_{21}a_{12} + b_{22}a_{22} \end{Vmatrix}$ formula bilan aniqlanadi. Ko`rsatish mumkinki $B \cdot A \neq A \cdot B$

Ushbu matrisalar berilgan bo`lsin:

$A = \begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix}$, va $B = \begin{Vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{Vmatrix}$, $C = A \cdot B$, $C = B \cdot A = \begin{Vmatrix} b_{11}a_{11} + b_{12}a_{21} & b_{11}a_{12} + b_{12}a_{22} \\ b_{21}a_{11} + b_{22}a_{21} & b_{21}a_{12} + b_{22}a_{22} \end{Vmatrix}$

Agar uchinchi tartibli

$$A = \begin{Vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{Vmatrix} \qquad B = \begin{Vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{Vmatrix}$$

Matrisalar berilgan bo`lsa, u holda $C = A \cdot B$ matrisa qo`yidagicha tuziladi:

$$C=\begin{bmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31}\cdot & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32}\cdot & a_{11}b_{13}+a_{12}b_{23}+a_{13}b_{33} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31}\cdot & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32}\cdot & a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33} \\ a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31}\cdot & a_{31}b_{12}+a_{32}b_{22}+a_{33}b_{32}\cdot & a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{33} \end{bmatrix}$$

1-Misol. $\begin{bmatrix} 3 & 1 & 1 \\ 2 & 3 & 0 \end{bmatrix}\cdot\begin{bmatrix} 1 & 1 \\ 3 & 2 \\ 2 & 0 \end{bmatrix}=\begin{bmatrix} 3\cdot1+1\cdot3+1\cdot2 & 2\cdot1+3\cdot3+0\cdot2 \\ 3\cdot1+1\cdot2+1\cdot0 & 2\cdot1+2\cdot3+0\cdot0 \end{bmatrix}=\begin{bmatrix} 8 & 11 \\ 5 & 8 \end{bmatrix}$

2-misol. $\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}\cdot\begin{bmatrix} 2 \\ 1 \end{bmatrix}=\begin{bmatrix} 4\cdot2+2\cdot1 \\ 3\cdot2+1\cdot1 \end{bmatrix}=\begin{bmatrix} 10 \\ 7 \end{bmatrix}$

Matrisalarni ko`paytirish ushbu $A(BC) \neq (AB)C$ o'rinli.

$$E=\begin{bmatrix} 1 & 0 & 0\ldots0 & 0 \\ 0 & 1 & 0\ldots0 & 0 \\ \ldots\ldots & & & \\ 0 & 0 & 0\ldots1 & 0 \\ 0 & 0 & 0\ldots0 & 1 \end{bmatrix}, \quad |E|=1, \quad \text{AE=EA=A}$$

2. Quyidagi matrisalar ko'paytmasunu topaylik

1. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & -1 & -3 \\ 4 & -2 & 3 & 1 \end{pmatrix}\begin{pmatrix} 3 & 5 \\ 1 & -4 \\ 6 & 2 \\ -3 & -1 \end{pmatrix}=$

$$=\begin{pmatrix} 1\cdot3+2\cdot1+3\cdot6+4\cdot(-3) & 1\cdot5+2\cdot(-4)+3\cdot2+4\cdot(-1) \\ 2\cdot3+1\cdot1+(-1)\cdot6+(-3)\cdot(-3) & 2\cdot5+1\cdot(-4)+(-1)\cdot2+(-3)\cdot(-1) \\ 4\cdot5+(-2)\cdot(-4)+3\cdot2+1\cdot(-1) & 4\cdot5+(-2)\cdot(-4)+3\cdot2+1\cdot(-1) \end{pmatrix}=$$

$$=\begin{pmatrix} 11 & -1 \\ 10 & 7 \\ 25 & 31 \end{pmatrix};$$

2. $\begin{pmatrix} 3 & 2 & 1 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ -3 & 2 \\ 2 & -3 \end{pmatrix}$ = (8, 4).

Mavzu bo`yicha takrorlash savollari

1. Qanday amallar matritsalar ustida chiziqli amallar deyiladi?
2. Ikkita matritsani hamma vaqt ko`paytirish mumkinmi?
3. Matritsalar ko`paytmasini aniqlash formulasini yozing?
4. Matritsalar ko`paytirilganda ularning o`rinlarini almashtirish mumkinmi?

3.2.4. Matritsa yordamida vektorni boshqa vektorga almashtirish

$X = x_1\bar{i} + x_2\bar{j} + x_3\bar{k}$ vektor berilgan bo`lsin. Uni $X = \begin{Vmatrix} x_1 \\ x_2 \\ x_3 \end{Vmatrix}$ matritsa shaklida yozamiz. Qo`yidagini bajaramiz

$$Y = A \cdot X = \begin{Vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{Vmatrix} \quad \begin{Vmatrix} x_1 \\ x_2 \\ x_3 \end{Vmatrix} = \begin{Vmatrix} a_{11}x_1 + & a_{12}x_2 + & a_{13}x_3 \\ a_{21}x_1 + & a_{22}x_2 + & a_{23}x_3 \\ a_{31}x_1 + & a_{32}x_2 + & a_{33}x_3 \end{Vmatrix} = \begin{Vmatrix} y_1 \\ y_2 \\ y_3 \end{Vmatrix}$$

$$\begin{cases} y_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ y_2 = a_{21}x_2 + a_{22}x_2 + a_{23}x_3 \\ y_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{cases}$$

Shunday qilib A matritsa yordamida X vektorni Y vektorga almashtirdik

Mavzu bo`yicha takrorlash savollari

1. Matritsa yordamida berilgan vektor boshqa vektorga qanday almashtiriladi**?**

3.2.5. Teskari matritsa tushunchasi

A matritsa yordamida X vektorni Y vektorga almashtiramiz Y=A·X. Agar Δ(A)≠0 bo'lsa, u vaqtda teskari almashtirish ham mavjud bo'ladi. A·X=Y==> X= A^{-1}Y= A^{-1}·AX=EX ==> X=EX. Endi berilgan matrisaga teskari matrisani topishni ko'rsatamiz.

$$A = \begin{Vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{Vmatrix} \text{ va } \Delta(A) = \begin{Vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{Vmatrix} \neq 0 \text{ bo`lsin}$$

Berilgan A matrisaga teskari A^{-1} matrisa quyidagi formula bilan topiladi. $A^{-1} = \begin{Vmatrix} \frac{A_{11}}{\Delta} & \frac{A_{21}}{\Delta} & \frac{A_{31}}{\Delta} \\ \frac{A_{12}}{\Delta} & \frac{A_{22}}{\Delta} & \frac{A_{32}}{\Delta} \\ \frac{A_{13}}{\Delta} & \frac{A_{23}}{\Delta} & \frac{A_{33}}{\Delta} \end{Vmatrix}$, bu erda A_{ij} lar a_{ij} elementlarning algebraik to'ldiruvchilaridir.

A^{-1} matrisa ta'rifga asosan shunday topilishi kerakki, toki $A \cdot A^{-1} = E = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix}$ bo'lsin.

Agar $\overline{A} = \begin{Vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{Vmatrix}$ deb olsak, u vaqtda $A^{-1} = \frac{1}{\Delta(a)} \cdot \overline{A}$ bo'ladi. Bunda $\overline{A}$ matrisa A matrisa bilan birlashtirilgan matrisa deyiladi.

3.3-§ Chipili tenglamalar tizimini matritsa usulida yozish va yechish

Faraz qilaylik A matritsa aniqlovchisi noldan farqli bo`lsin, $\Delta(A)\neq 0$. $A\cdot X=D$ dan

$A^{-1}\cdot AX=A^{-1}\cdot D \rightarrow X=A^{-1}\cdot D$, $X = \frac{1}{\Delta(A)} \cdot \overline{A} \cdot D$ (3) ekani kelib chiqadi. (3) dan

$$\begin{Vmatrix} x_1 \\ x_2 \\ x_3 \end{Vmatrix} = \frac{1}{\Delta(A)} \begin{Vmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{Vmatrix} \cdot \begin{Vmatrix} d_1 \\ d_2 \\ d_3 \end{Vmatrix} = \frac{1}{\Delta(A)} \cdot \begin{Vmatrix} A_{11}d_1 + A_{21}d_2 + A_{31}d_3 \\ A_{12}d_1 + A_{22}d_2 + A_{32}d_3 \\ A_{13}d_1 + A_{23}d_2 + A_{33}d_3 \end{Vmatrix}$$

ekani kelib chiqadi. Bulardan

$x_1 = \frac{A_{11}d_1 + A_{21}d_2 + A_{31}d_3}{\Delta(A)}, x_2 = \frac{A_{12}d_1 + A_{22}d_2 + A_{32}d_3}{\Delta(A)}, x_3 = \frac{A_{13}d_1 + A_{23}d_2 + A_{33}d_3}{\Delta(A)}$ shaklda

yozish mumkin. Bular esa ma'lum Kramer formulalaridir:

Misol. Tenglamadan X matrisani toping va tekshirish o'tkzing.

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 5 & -2 \\ 0 & -1 & 5 \end{pmatrix} X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$x_1 = \frac{\begin{vmatrix} d_1 & a_{12} & a_{13} \\ d_2 & a_{22} & a_{23} \\ d_3 & a_{23} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{23} & a_{33} \end{vmatrix}}, \; x_2 = \frac{\begin{vmatrix} a_{11} & d_1 & a_{13} \\ a_{21} & d_2 & a_{23} \\ a_{31} & d_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{23} & a_{33} \end{vmatrix}}, \; x_3 = \frac{\begin{vmatrix} a_{11} & a_{12} & d_1 \\ a_{21} & a_{22} & d_2 \\ a_{31} & a_{23} & d_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{23} & a_{33} \end{vmatrix}}.$$

Namunaviy misol va ko'rsatmalar

$$\begin{cases} 3x_1 - 2x_2 + 4x_3 = 4, \\ 2x_1 + 4x_2 - 5x_3 = -15, \\ -7x_1 + x_2 - 8x_3 = -9, \end{cases}$$

tenglamalar sistemasi teskari matritsalar usullari yordamida yechilsin. Berilgan tenglamalar sistemasini $A \cdot X = B$ ko'rinishida yozib olamiz. Bu erda

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Agar A matritsaning determinanti $\Delta \neq 0$ bo'lsa, u holda $A \cdot A^{-1} = E$ tenglikni kanoatlantiruvchi A^{-1} matritsa A matritsaga teskari matritsa deyiladi. Bu erda E-birlik matritsa, ya'ni $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Agar A matritsaga teskari bo'lgan A^{-1} matritsa mavjud bo'lsa, u holda berilgan sistemaning yechimi $X = A^{-1} \cdot B$ formula yordamida aniqlanadi. A^{-1} matritsani aniqlash jarayoni quyidagi algoritmga ega:

1) Berilgan A matritsaning determinanti hisoblanadi

$$\Delta = \det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$$

2) A matritsaning a_{ij} elementlarining $A_{ij}=(-1)^{i+j}M_{ij}$ (i,j=1,2,3), ko'rinishda topib, bu erda A_{ij} – a_{ij} elementning algbraik to'ldiruvchisi, M_{ij}– a_{ij} elementning minori,

$$\tilde{A} = \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$

matritsa hosil qilinadi.

3) Teskari matritsani topish

$$A^{-1}=\frac{1}{\Delta}\cdot\tilde{A}=\begin{pmatrix}\frac{A_{11}}{\Delta} & \frac{A_{21}}{\Delta} & \frac{A_{31}}{\Delta}\\ \frac{A_{12}}{\Delta} & \frac{A_{22}}{\Delta} & \frac{A_{32}}{\Delta}\\ \frac{A_{13}}{\Delta} & \frac{A_{23}}{\Delta} & \frac{A_{33}}{\Delta}\end{pmatrix}$$

formula orqali ifodalanadi. Demak, yuqorida qayd etilganidek, dastlab,

$A=\begin{pmatrix}3 & -2 & 4\\ 2 & 4 & -5\\ -7 & 1 & -8\end{pmatrix}$ matritsaga teskari bo'lgan A^{-1} matritsani aniqlaymiz. Bizga ma'lumki, $\Delta=\begin{vmatrix}3 & -2 & 4\\ 2 & 4 & -5\\ -7 & 1 & -8\end{vmatrix}=-63\neq 0$

$\tilde{A}$ matritsani tuzish uchun $A_{ij}(i,j=1,2,3)$ algebraik to'ldiruvchilarni aniqlaymiz.

$$A_{11}=(-1)^{1+1}\begin{vmatrix}4 & -5\\ 1 & -8\end{vmatrix}=-27,\quad A_{23}=(-1)^{2+3}\begin{vmatrix}3 & -2\\ -7 & 1\end{vmatrix}=11,$$

$$A_{12}=(-1)^{1+2}\begin{vmatrix}2 & -5\\ -7 & -8\end{vmatrix}=51,\quad A_{31}=(-1)^{3+1}\begin{vmatrix}-2 & 4\\ 4 & -5\end{vmatrix}=-6,$$

$$A_{13}=(-1)^{1+3}\begin{vmatrix}2 & 4\\ -7 & 1\end{vmatrix}=30,\quad A_{32}=(-1)^{3+2}\begin{vmatrix}3 & 4\\ 2 & -5\end{vmatrix}=23,$$

$$A_{21}=(-1)^{2+1}\begin{vmatrix}2 & 4\\ 1 & -8\end{vmatrix}=-12,\quad A_{33}=(-1)^{3+3}\begin{vmatrix}3 & -2\\ 2 & 4\end{vmatrix}=16$$

$$A_{22}=(-1)^{2+2}\begin{vmatrix}3 & 4\\ -7 & -8\end{vmatrix}=4$$

Izlangan teskari matritsa $A^{-1}=\frac{1}{\Delta}\cdot\tilde{A}=\frac{1}{-63}\begin{pmatrix}-27 & -12 & -6\\ 51 & 4 & 23\\ 30 & 11 & 16\end{pmatrix}$ ko'rinishga ega bo'ladi.

Shunday qilib, $X=A^{-1}\cdot B$ formulaga asosan

$$X=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\frac{1}{-63}\begin{pmatrix}-27 & -12 & -6\\ 51 & 4 & 23\\ 30 & 11 & 16\end{pmatrix}\cdot\begin{pmatrix}4\\ -15\\ -9\end{pmatrix}=$$

$$= \frac{1}{-63}\begin{pmatrix} -27\cdot 4+(-12)\cdot(-15)+(-6)\cdot(-9) \\ 51\cdot 4+4\cdot(-15)+23\cdot(-9) \\ 30\cdot 4+11\cdot(-15)+16\cdot(-9) \end{pmatrix} = \frac{1}{-63}\begin{pmatrix} 126 \\ -63 \\ -189 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$$

ni topamiz. Demak, $x_1 = -2,\ x_2 = -1,\ x_3 = -3$ ikkala usulda ham bir xil yechimni olamiz. Javob: $\{-2,1,3\}$

Mavzu bo`yicha takrorlash savollari

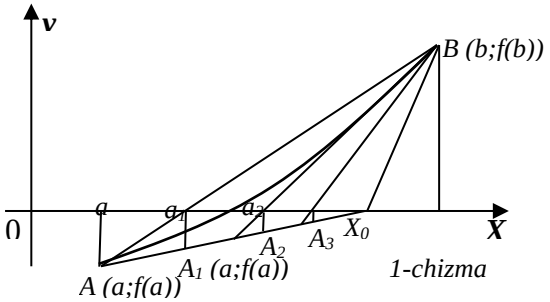
1. Chiziqli tenglamalar tizimini qanday qilib matritsa yordamida yozish mumkin va echish mumkin?
2. Bunday echish hamma vaqt mumkinmi?

3.4-§ Tenglamaning haqiqiy ildizlarini taqribiy hisoblash

Ma'lumki tenglamalarning ildizlarini hamma vaqt ham aniq topib bo'lmaydi. Agar tenglama koeffitsientlari berilgan bo'lsa, uning ildizlarini istagan darajada aniqlik bilan taqribiy hisoblash mumkin. Quyida tenglama ildizlarini taqribiy hisoblashning ba'zi usullarini bayon qilamiz.

3.4.1. Vatarlar usuli

Ushbu $f(x)=0$ (1) tenglama berilgan bo'lib $f(x)$ funktsiya $[a,b]$ kesmada uzluksiz va ikki marta differentsiallanuvchi bo'lsin. Bu funktsiyaning $[a,b]$ kesma chetlaridagi qiymatlari har xil ishorali bo'lsin, ya'ni $f(a)\cdot f(b)<0$. Aniqlik uchun $f(a)<0$ va $f(b)>0$ deb faraz qilamiz. $y=f(x)$ funktsiya $[a,b]$ kesmada uzluksiz bo'lgani uchun uning grafigi a va b nuqtalar orasidagi birorta nuqtada Ox o'qni kesadi. Bu nuqta abtsissasi $f(x)=0$ tenglamaning ildizlaridan biridir. Shu ildizni topamiz. Buning uchun AB vatarni o'tkazamiz. (1-rasm). Bu vatarning Ox o'qi bilan kesishgan nuqtasining A B abtsissasi ildizning taqribiy qiymati bo'ladi. AB vatar tenglamasini yozamiz:



1-chizma

$$\frac{y-f(a)}{f(b)-f(a)}=\frac{x-a}{b-a}$$

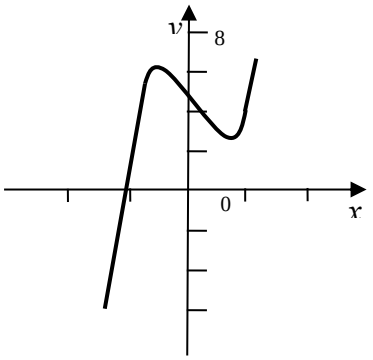
$x=a_1$ bo'lganda, $y=0$ bo'lgani uchun $\frac{-f(a)}{f(b)-f(a)}=\frac{a_1-a}{b-a}$

bundan $a_1=a-f(a)\frac{b-a}{f(b)-f(a)}$ yoki $a_1=\frac{af(b)-bf(a)}{f(b)-f(a)}$ (2) ekanligini topamiz.

Ildizning aniqroq qiymatini topish uchun $f(a_1)$ qiymatini aniqlaymiz. Agar $f(a_1)<0$ bo'lsa, (2) formulani $[a,b]$ kesmaga qo'llanib, shu usulni takrorlaymiz. Agar $f(a_1)>0$ bo'lsa, shu formulani $[a,a_1]$ kesmaga qo'llanamiz. Bu usulni bir necha marta takrorlab, ildizning yanada aniqroq a_2, a_3 va x.k. qiymatlarini topamiz.

Misol: $f(x)=x^3-3x+5=0$ tenglamaning yechimi vatar usuli bilan topilsin.

Yechish: $f(x)=x^3-3x+5$ funktsiyaning grafigi Ox o'qini $[-2,5;-2]$ kesmada kesib o'tadi. (2-rasm). Bu erda



2- chizma

$a=-2,5;\ b=-2;\ f(a)=f(-2,5)=-3,125<0;\ f(b)=f(-2)=3>0$

$f(a)\cdot(b)=f(-2,5)\cdot f(-2)<0$

2-formulaga asosan:

$$a_1=-2,5-\frac{[-2-(-2,5)]\cdot(-3,125)}{3-(-3,125)}=-2,5+\frac{0,5\cdot3,125}{6,125}\approx-2,2449$$

Endi $f(a_1)$ ni hisoblaymiz

$$f(a_1)=f(-2,2449)=(-2,2449)^3-3(-2,2449)+5=0,4214>0$$

Demak, tenglamaning ildizi $[-2,5;-2,24]$ da yotadi, chunki

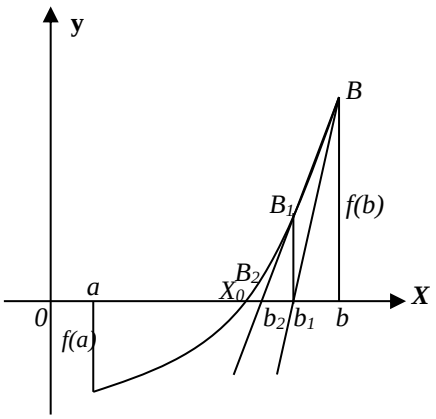
$$f(a)=f(-2,5)=-3,125<0$$
$$f(a_1)=f(-2,2449)=0,4214>0$$

va (2) formulaga asosan:

$$a_2=-2,5-\frac{[-2,2449-(-2,5)]\cdot(-3,125)}{0,4214-(-3,125)}=-2,5+\frac{0,2551\cdot3,125}{3,5464}\approx-2,2752$$

Demak, tenglamaning ildizi deb $x_0 \approx -2{,}2752$ sonni olsa bo'ladi.

Bu erda ham vatar usuldagi kabi $f(a)<0$, $f(b)>0$ va $[a,b]$ kesmada birinchi hosilaning ishorasi o'zgarmaydi deb faraz qilamiz. Bu xolda $[a,b]$ kesmada $f(x)=0$ tenglama bitta ildizga ega bo'ladi. $[a,b]$ kesmani $f(x)$ funktsiyaning ikkinchi xosilasi o'z ishorasini o'zgartirmaydigan darajada kichik qilib olsin, u vaqtda funktsiya grafigi bu kesmada yo faqat qavariq yo faqat botiq bo'ladi.

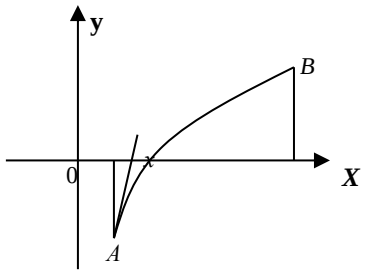


3-chizma

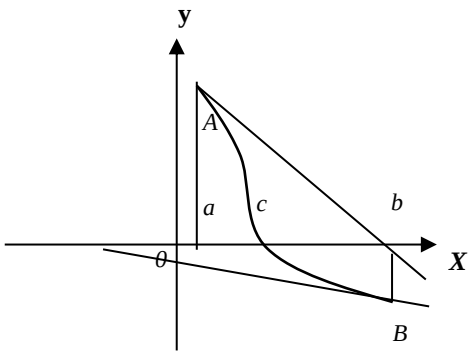
Egri chiziqqa B nuqtada urinma o'tkazamiz (3-rasm). Urinmaning *Ox* o'q bilan kesishgan nuqtasining abtsissasini b_1 bilan belgilasak, u ildizning taqribiy qiymati bo'ladi. Shu abtsissasini topish uchun B nuqtadagi urinma tenglamasini yozamiz. $y-f(b)=f^1(b)(x-b)$. Agar $x=b_1$, desak $y=0$ bo'ladi va

$$-f(b)=f^1(b)(b_1-b),\quad b_1=b-\frac{f(b)}{f^1(b)} \qquad (3)$$

bo'ladi. So'ngra b_1 nuqtada Ox o'qiga perpendikulyar to'g'ri chiziq o'tkazib $y=f(x)$ funktsiya grafigi bilan uning kesishish nuqtasi B_1 ni topamiz. Endi $_1$ nuqtada $y=f(x)$ funktsiya grafigiga urinma o'tkazib berilgan tenglama ildizining aniqroq qiymati b_2 ni topamiz. Bu usulni bir necha marta takrorlab, ildizning istalgan aniqlikdagi tarqibiy qiymatini topa olamiz. Rasmdan ko'rinib turibdiki, yoyning qaysi chegara nuqtasida $f(x),\ f'(x)\ va\ f''(x)$ larning ishoralari bir xil bo'lsa, o'sha nuqtada $y=f(x)$ egri chiziqqa urinma o'tkazish kerak. Bunday nuqta hamma vaqt mavjuddir, chunki $f(x)$ funktsiya $[a,b]$ kesma



4- chizma



5- chizma

chekkalarida har xil ishorali qiymatlar qabul qiladi. $f''(x)$ esa o'z ishorasini o'zgartirmaydi. Bu qoida $f'(x)<0$ bo'lganda ham o'z kuchini saqlaydi. Agar urinma intervalning chap chegarasidan o'tkazilsa, (3) formulada b urniga a ni quyish kerak: $a_1 = a - \dfrac{f(a)}{f(a)}$ (3)

Agar (a, b) interval C burilish nuqtasi bo'lsa, urinmalar usuli ildizning (a, b) intervaldan tashqarida yotgan taqribiy qiymatini berishi mumkin (5-rasm).

Misol: $f(x) = x^3 - 3x + 5 = 0$ tenglamaning yechimi urinma usuli bilan topilsin.

Yechish: 1-yoki 2-rasmdan foydalanib berilgan tenglamaning ildizi $[-2{,}5;\ -2]$ kesmada yotishini aniqladik. Endi urinmani qaysi nuqtada utkazishni aniqlaylik:

$$f`(x) = 3x^2 - 3, \qquad f`(x) = 6x \qquad \left.\begin{array}{l} f(2{,}5) = -3{,}125 < 0 \\ f``(-2{,}5) = 6(-2{,}5) = -15 < 0 \end{array}\right\} =>$$

$$\left.\begin{array}{l} f`(x) < 0 \\ f``(x) < 0 \end{array}\right. \qquad \left.\begin{array}{l} f(-2) = 3 > 0 \\ f``(-2) = 6(-2) = -12 < 0 \end{array}\right\} => \qquad \begin{array}{l} f(x) > 0 \\ f``(x) < 0 \end{array}$$

Urinmani A (-2,5; -3,125) nuqtada o'tkazamiz, (3) formulaga asosan

$b_1 = -2{,}5 - \dfrac{-3{,}125}{19{,}75} = -2{,}3016 \qquad b_2 = b_1 - \dfrac{f(b_1)}{f``(b_1)}$ larni hisoblaymiz.

$$f(b_1) = f(-2{,}3016) = (-2{,}3016)^3 - 3(-2{,}3016) + 5 = -0{,}267$$

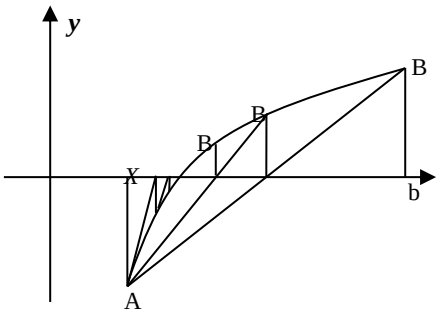
$$f`(b_1) = f`(-2.3016) = 3(-2{,}3016)^2 - 3 = 12{,}87$$

$b_2 = -2{,}3016 - \dfrac{-0{,}267}{12{,}87} = -2{,}3016 + 0{,}02074 = -2{,}2808$ bo'lar ekan.

3.4.2. Vatar va urinmalar yoki birlashtirilgan usul

Vatarlar usuli va urinmalar usulini $[x_1, x_2]$ kesmada bir vaktda qo'llanib, izlangan x ildizning ikki tomonida yotgan a_1 va b_1 nuqtalarni topamiz (chunki $f(a_1)$ va $f(b_1)$ ordinatalarning ishoralari har xil) (6-rasm). So'ngra $[a_1, b_1]$ kesmada yana vatarlar usulini va urinmalar usulini qo'llanamiz. Natijada ildizning qiymatiga yanada yaqinroq ikkita a_2 va b_2 sonlarni topamiz. Topilgan taqribiy qiymatlar orasidagi ayirma talab etilgan aniqlik darajasidan qichiq bo'lguncha ishni shunday davom ettiramiz.

Birlashtirilgan usul bilan biz izlanayotgan ildizga bir vaqtda ikki tomondan yaqinlashamiz (ya'ni bir vaqtda ildizning haqiqiy qiymatidan ortiqroq va kamroq taqribiy qiymatlarni topamiz).



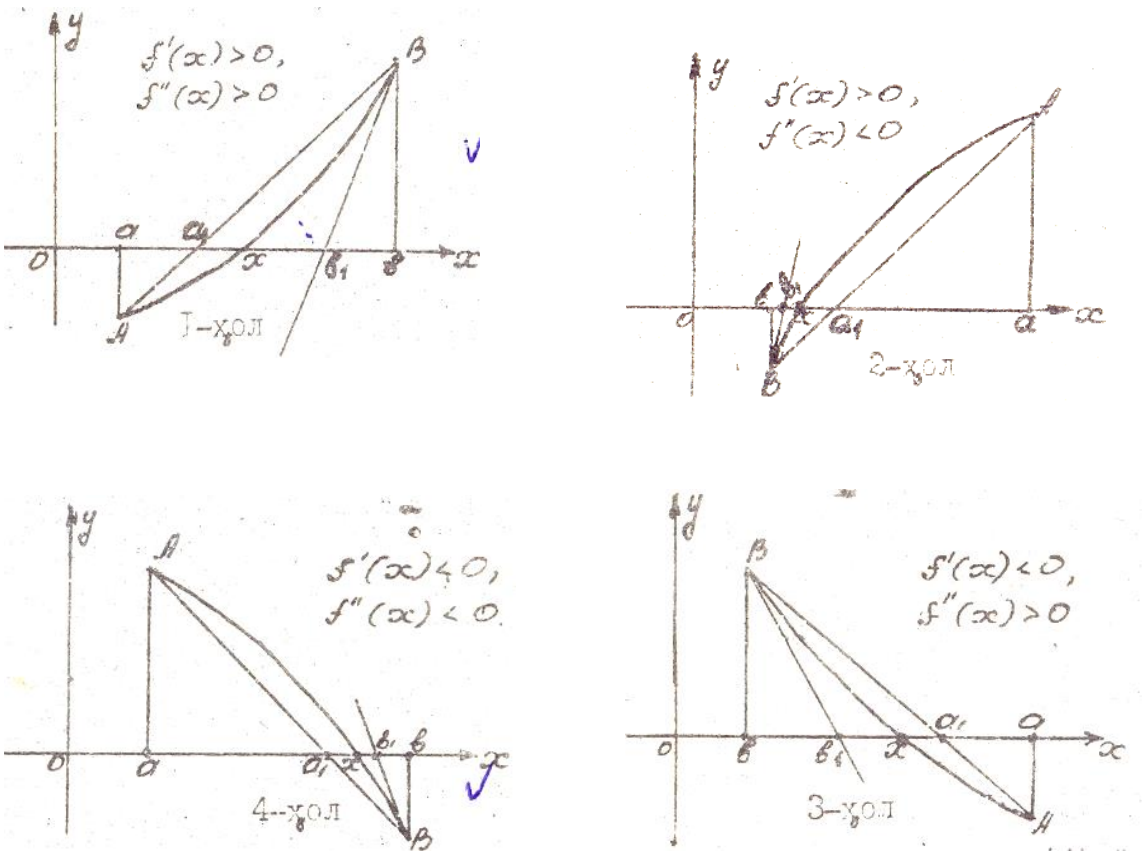
6-chizma

Vatarlar usuli va urinmalar usullarini berilgan aniqlikka ega bo'lmagancha biror n marta takrorlasak, (2) va (3) formulalarning ko'rinishi quyidagicha bo'ladi.

$$a_n = a_{n-1} - f(a_{n-1})\frac{b_{n-1} - a_{n-1}}{f(b_{n-1}) - f(a_{n-1})} \qquad (4)$$

$$b_n = b_{n-1} - \frac{f(b_{n-1})}{f`(b_{n-1})} \qquad (5)$$

Agar $[a,b]$ kesmada $f(a)\cdot f(b) < 0$ bo'lib, $f'(x)$ $f''(x)$ xosilalar o'z ishorasini saqlasa, quyidagi to'rt hol bo'lishi mumkin (1-hol, 2-hol, 3-hol, 4-hol deb nomerlangan shakllarga qarang).1-hol, 2-hol, 3-hol, 4-hol



7 - *chizma*

1- va 4-hollarda (4) formulani chap chegaraviy nuqta uchun (5) formulani esa ung chegaraviy nuqta uchun qo'llaymiz.

2- va 3-hollarda esa (4) formulani o'ng chegaraviy nuqta uchun, (5) formulani esa chap chegaraviy nuqta uchun qo'llaymiz.

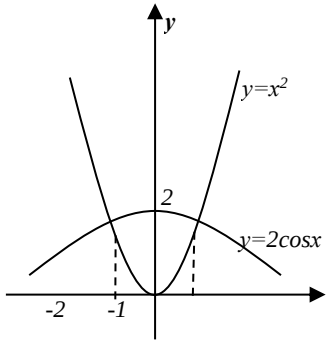
Misol: $f(x)=x^2-2\cos x=0$ tenglama berilgan bo'lsin. Uning musbat ildizi 0,0001 aniqlikda topilsin.

Yechish: Berilgan $x^2-2\cos x=0$ tenglamani ikki funktsiyaning tengligi ko'rinishida yozamiz: $x^2=2\cos x$ *va* $\varphi(x)=x^2, \psi(x)=2\cos x$ deb belgilab olamiz. $\varphi(x)=x^2$ va $\psi(x)=2\cos x$ funktsiyalarning grafiklarini chizib, ularning kesishish nuqtalari abstsissalarini belgilaymiz. Ikkita ildizga ega bo'lamiz: biri – musbat, ikkinchisi – manfiydir. (8-rasm).

Musbat ildiz [1;1,3] kesmada yotadi. Bu kesma uchun quyidagi hollarni qaraymiz.

1) [1; 1,3] kesma uchlaridagi nuqtalarda $f(x)$ funktsiyaning qiymatlari ishoralarini aniqlaymiz:

$$f(1)=1^2-2\cos 1=1-2\cdot 0{,}5403=-0{,}0806<0,$$
$$f(1{,}3)=1{,}3^2-2\cos 1{,}3=1{,}69-2\cdot 0{,}2675=1{,}155>0$$



8- chizma

$f(1)\cdot f(1{,}3)<0$ bo'lgani uchun $[1;\,1{,}3]$ da $f(x)=0$ tenglamaning ildizi bor.

2) $f(x)=x^2-2\cos x$ funktsiya $(-\infty;+\infty)$ da uzluksiz, hamda $[1;\,1{,}3]$ da birinchi va ikkinchi hosilalarga ega. Ular

$$f'(x)=2x+2\sin x,\quad f''(x)=2+2\cos x$$

ko'rinishda bo'ladi.

3) $x=1$ va $x=1{,}3$ da birinchi va ikkinchi xosilalarning ishoralarini aniqlaymiz.

$$f`(1)=2\cdot 1+2\sin\ b_1=2+2\cdot 0{,}8415=3{,}6830>0,$$
$$f`(1{,}3)=2\cdot 1{,}3+2\sin 1{,}3=2{,}6+2\cdot 0{,}9636=4{,}5272>0,$$
$$f``(1)=2+2\cos 1=2+2\cdot 0{,}5403=3{,}0806>0,$$
$$f``(1{,}3)=2+2\cos 1{,}3=2+2\cdot 0{,}2675=2+0{,}5350=2{,}5350>0$$

Demak, $f'(x)>0,\ f''(x)>0$, bu esa 7-rasmdagi 1-hol bo'lib, $a=1$ va $b=1{,}3$ bo'lar ekan.

a_1 va b_1 larni (4) va (5) formulalar yordamida topimiz.

$$a_1=a-f(a)\frac{b-a}{f(b)-f(a)}=1-f(1)\cdot\frac{1{,}3-1}{f(1{,}3)-f(1)}=1-(-0{,}0806)\frac{0{,}3}{1{,}155-(-0806)}\approx 1{,}01957$$

$$b_1=b-\frac{f(b)}{f`(b)}=1{,}3-\frac{f(1{,}3)}{f`(1{,}3)}=1{,}3-\frac{1{,}155}{4{,}5272}\approx 1{,}04488$$

Ammo $\Delta_1=|a_1-b_1|=|1{,}01957-1{,}04488=0{,}02531>0{,}0001|$ bo'lgani uchun bu topilgan qiymatlar izlanayotgan ildizning hali talab qilingan darajadagi aniqlikda topilmaganini ko'rsatadi.

Shuning uchun $f(a_1),\ f(b_1)\ va\ f'(b_1)$ larni hisoblaymiz.

$$f(a_1) = f(1{,}01957) = (1{,}01957)^2 - 2\cdot\cos 1{,}01957 = 1{,}03952 - 2\cdot 1{,}0474 \approx 0{,}00798;$$

$$f(b_1) = f(1{,}04488) = (1{,}04488)^2 - 2\cdot\cos 1{,}04488 \approx 0{,}087758;$$

$$f`(b_1) = f`(1{,}04488) = 2\cdot 1{,}04488 + 2\cdot\sin 1{,}04488 \approx 3{,}819488$$

topilgan $f(a_1),\ f(b_1),\ f'(b_1)$ lardan foydalanib, a_2 va b_2 larni topamiz.

$$a_2 = a_1 - f(a_1)\cdot\frac{b_1 - a_1}{f(b_1) - f(a_1)} = 1{,}01957 - (-0{,}007944)\cdot\frac{1{,}04488 - 1{,}01957}{0{,}087758 - (-0{,}007944)} = 1{,}01957 + \frac{0{,}007544\cdot 0{,}02531}{0{,}095702} \approx 1{,}02167$$

$$b_2 = b_1 - \frac{f(b_1)}{f`(b_1)} = 1{,}04488 - \frac{0{,}087758}{3{,}819488} = 1{,}02190, \quad \Delta_2 = |a_2 - b_2| = |1{,}02167 - 1{,}02190| = 0{,}00023 > 0{,}0001$$

Talab qilingan aniqlik hosil bo'lmadi. Shuning uchun hisoblashlarni davom ettiramiz.

$$f(a_2) = f(1{,}02157) \approx -0{,}0020765;\ f(b_2) = f(1{,}02190) \approx 0{,}000796;\ f`(b_2) = f`(1{,}02190) \approx 3{,}750002$$

$$a_3 = a_2 - f(a_2)\frac{b_2 - a_2}{f(b_2) - f(a_2)} = 1{,}02167 - (-0{,}0020765)\cdot\frac{1{,}02190 - 1{,}02167}{0{,}0007976 - (-0{,}0020765)} \approx 1{,}0218097$$

$$b_3 = b_2 - \frac{f(b_2)}{f`(b_2)} = 1{,}02190 - \frac{0{,}0007976}{3{,}750002} \approx 1{,}0218074;$$

$$\Delta_3 = |a_3 - b_3| = |1{,}0218097 - 1{,}0218074| = 0{,}0000023 < 0{,}0001$$

Shunday qilib, $f(x) = x^2 - 2\cos x$ tenglamaning 0,0001 aniqlikdagi taqribiy ildizi $x \approx \frac{1}{2}(1{,}0218074 + 1{,}0218097) = 1{,}02180085$ ga taqriban teng bo'lar ekan

3.5-§ Chiziqli tenglamalar sistemasini yechishning Gauss usuli.

n ta no'malumli m ta tenglamalar sistemasini qaraymiz.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{cases} \qquad (1)$$

Agar chiziqli tenglamar sistemasi echimga ega bo'lsa, u birgalikda, agar echimga ega bo'lmasa, u birgalikda emas deyiladi. quyidagi elementar

almashtirishlar natijasida tenglamalar sistemasi o'ziga teng kuchli sistemaga almashadi.

1) Istalgan ikki tenglamani urinlarini almashtirilsa;
2) Tenglamalardan istalgan birini ikkala tomonini noldan farkli songa kupaytirilsa;
3) Tenglamalardan birini istalgan xakikiy songa kupaytirib, boshqa tenglamaga qo'shilsa.

Agar n>m bo'lsa, n-m ta bir xil noma'lumli hadlarni tengliklarning o'ng tomoniga olib o'tib, o'ng tomidagi nomalumlarga ixtiyoriy qiymatlarni qabul qiladi deb, tenglamalar sistemasini n=m holga keltirib olish mumkin. Shuni e'tiborga olib, (1) sistemani $n=m$ holi uchun echamiz.

Gauss usulining mohiyati noma'lumlarni ikkinchi tenglamadan boshlab, ketma-ket yo'qotib oxirgi teglamada bitta no'malum qolguncha davom ettiriladi va oxirgi tenglamadan yuqoriga qarab no'malumlarni ketma-ket topib, echim hosil qilinadi.

1-qadam. (1) sistemada birinchi tenglamani har ikki tomonini a_{11} ga bo'lib, teng kuchlik ushbu sistemani hosil qilamiz:

$$\begin{cases} x_1 + \left(\frac{a_{12}}{a_{11}}\right)x_2 + \ldots + \left(\frac{a_{1n}}{a_{11}}\right)x_n = \frac{b_1}{a_{11}} \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n \end{cases} \qquad (2)$$

Birinchi tenglamani a_{21} ga ko'paytirib ikkinchi tenglamadan, a_{31} ga k''paytirib uchinchi tenglamadan va hokazo a_{n1} ga ko'paytirib, n-tenglamadan ayiramiz. Natijada yana berilgan sistemaga teng kuchli ushbu yangi sistemani hosil qilamiz:

$$\begin{cases} x_1 + a'_{12}\,x_2 + \ldots + a'_{1n}\,x_n = b'_1 \\ a'_{22}\,x_2 + \ldots + a'_{2n}\,x_n = b'_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n2}x_2 + \ldots + a'_{nn}\,x_n = b'_n \end{cases} \qquad (3)$$

Bu sistemada quyidagicha belgilashlar kiritilgan:

$$a'_{1k} = a_{1k}/a_{11},\ a'_{ik} = a_{ik} - (a_{1k}/a_{11})a_{i1},$$

$$b'_1 = b_1/a_{11},\ b'_i = b_i - (b_1/a_{11})a_{i1}.$$

$$i=2,..,n$$

$$k=2,..,n.$$

Agar (3) sistemada biror tenglama chap tomonnidagi barcha koeffitsentlari nolga teng, o'ng tomoni esa noldan farqli bo'lsa, ya'ni

$$0x_2 + 0x_3 + ... + 0x_n = b_k \quad (4)$$

ko'rinishdagi tenglama hosil bo'lsa, sistema birgalikda emas deyiladi va ishni shu erda to'xtaliladi.

Agar (4) ko'rinishdagi tenglama hosil bo'lmasa keyingi qadamga o'tiladi.

2-qadam. Ikkinchi tenglamani a_{22} koefitsentga bo'lamiz, hosil bo'lgan sistemaning ikkinchi tenglamasini ketma-ket $a'_{32},...,a'_{n2}$ larga ko'paytirib uchinchi, to'rtinchi va hokazo tenglamalardan ayiramiz.

Biz bu jarayonni oxirgi tenglamada x_n noma'lum qolguncha dovom ettirsak, dastlabki sistemaga teng kuchli

$$\begin{cases} x_{11} + c_{12}x_1 + \ldots + c_{1n}x_n = d_1 \\ x_2 + \ldots + c_{2n}x_n = d_2 \\ x_{n-1} + c_{n-1n}x_n = d_{n-1} \\ x_n = d_n \end{cases} \quad (5)$$

ko'rinishdagi sistemaga ega bo'lamiz. $x_n=d_n$ qiymatini (n-1) tenglama qo'yib x_{n-1} ni topamiz va hokazo, bu ishni x_1 topilguncha davom ettiramiz.

3.6-§ To'rt no'ma'lumli to'rta chiziqli algebraik tenglamalar sistemasini Gauss usuli bilan yechish

Ushbu chiziqli to`rt no'malumli to`rtta tenglamalar sistemasi berilgan bo`lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = a_{15} \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = a_{25} \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 = a_{35} \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = a_{45} \end{cases} \quad (1)$$

Faraz qilaylik $a_{11} \neq 0$ (yetakchi element) bo`lsin (agar $a_{11} = 0$ bo`lsa x_1 no'malum oldidagi koeffisienti nol bo`lmagan tenglama bilan birinchi tenglamani almashtiramiz).

Birinchi tenglamaning barcha koyefetsientlarini a_{11} ga bo`lsak:

$$\begin{aligned} &x_1 + \frac{a_{12}}{a_{11}} x_2 + \frac{a_{13}}{a_{11}} x_3 + \frac{a_{14}}{a_{11}} x_4 = \frac{a_{15}}{a_{11}} \\ &x_1 + b_{12}^{(1)} x_2 + b_{13}^{(1)} x_3 + b_{14}^{(1)} x_4 = b_{15}^{(1)} \end{aligned} \qquad (2)$$

hosil bo`ladi, bu yerda $b_{1j}^{(1)} = \frac{a_{1j}}{a_{11}}$ $(j \geq 2)$

Endi (2) ni a_{21} ga ko`paytirib (1) sistemaning ikkinchi tenglamasidan, (2) ni $a_{31}$ ga ko`paytirib, o`sha sistemaning uchunchi tenglamasidan va nihoyat (2) ni $a_{41}$ ga ko`raytirib (1) sistemaning to`rtinchi tenglamasidan ayirib, quyidagi tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} a_{22}^{(1)} x_2 + a_{23}^{(1)} x_3 + a_{24}^{(1)} x_4 = a_{25}^{(1)} \\ a_{32}^{(1)} x_2 + a_{33}^{(1)} x_3 + a_{34}^{(1)} x_4 = a_{35}^{(1)} \\ a_{42}^{(1)} x_2 + a_{43}^{(1)} x_3 + a_{44}^{(1)} x_4 = a_{45}^{(1)} \end{cases} \qquad (3)$$

bu yerda $a_{ij}^{(1)} = a_{ij} - a_{i1} \cdot b_{1j}^{(1)}$ $,(i, j \geq 2).$ (3) sistemaning yetakchi elementi $a_{22}^{(1)} \neq 0$ deb faraz qilib, birinchi tenglamaning barcha koeffisentlarini $a_{22}^{(1)}$ gab bo`lib, quyidagini hosil qilamiz:

$x_2 + b_{a3}^{(2)} x_3 + b_{2\cdot 4}^{(2)} = b_{25}^{(2)};$ (4) bu yerda $b_{2j}^{(2)} = \frac{a_{2j}^{(1)}}{a_{22}^{(1)}}, (j \geq 3)$

Endi (4) ni ketma-ket $a_{32}^{(1)}, a_{42}^{(1)}$ larga ko`paytirib, (3) sistemaning ikkinchi va uchunchi tenglamalaridan ayiramiz.

$$\begin{cases} a_{33}^{(2)} x_3 + a_{34}^{(2)} x_4 = a_{35}^{(2)} \\ a_{43}^{(2)} x_3 + a_{44}^{(2)} x_4 = a_{45}^{(2)} \end{cases} \qquad (5)$$

bu yerda $a_{ij}^{(2)} = a_{ij}^{(1)} - a_{i2}^{(1)} \cdot b_{2j}^{(2)}, (ij \geq 3)$ (5) ning birinchi tenglamasida $a_{33}^{(2)} \neq 0$ deb

(yetakchi element) faraz qilib birinchi tenglamaning barcha elementlarini $a_{33}^{(2)}$ ga bo`lsak: $x_3 + b_{34}^{(3)} x_4 = b_{35}^{(3)}$ (6) tenglama hosil bo`ladi, bu yerda $b_{3j}^{(2)} = \frac{a_{3j}^{(2)}}{a_{33}^{(2)}}, (j \geq 4)$

(6) ni $a_{43}^{(2)}$ ga ko`paytirib (5) ning ikkinchi tenglamasidan ayirsak $a_{44}^{(3)} x_4 = a_{45}^{(3)}$ (7) hosil bo`ladi, bu yerga $a_{ij}^{(3)} = a_{ij}^{(2)} - a_{i3}^{(2)} \cdot b_{3j}^{(3)}, (i, j \geq 4)$ x_4 ni (7) tenglikdan topamiz:

$x_4 = \frac{a_{45}^{(3)}}{a_{44}^{(3)}} = b_{45}^{(4)}$ (6), (4), (2) lardan foydalanib x_3, x_2, x_1 – larni topamiz:

$$\left.\begin{aligned} x_3 &= b_{35}^{(3)} - b_{34}^{(3)} \cdot x_4 \\ x_2 &= b_{25}^{(2)} - b_{24}^{(2)} x_4 - b_{23}^{(2)} x_3 \\ x_1 &= b_{15}^{(1)} - b_{14}^{(1)} x_4 - b_{13}^{(1)} x_3 - b_{12}^{(1)} x_2 \end{aligned}\right\} \qquad (8)$$

Tenglamalar sistemasining yuqorida ko`rsatilgan usul bilan yechilishi, ya’ni navbat bilan $x_1$, $x_2$, $x_3$ noma’lumlarni yo`qotib boorish usuli bilan yechilishi, tenglamalar sistemasini yechishning Gauss usuli deb ataladi.

Quyida tenglamalar sistemasini Gauss usuli bilan yechish sXemasini ko`rsatamiz. Hisoblashda xatoga yo`l qo`ymaslik uchun hisoblash jarayonini nazorat qilish ma’quldir. Buning uchun har bir satrning elementlari yig`indisi topiladi.

x_1	x_2	x_3	x_4	Ozod had	$\sum$	Formulalar	Sistema bo`limi
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}		1 A
a_{21}	a_{22}	a_{23}	a_{24}	A_{25}	A_{26}		
a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}		
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}		
1	$b_{12}^{(1)}$	$b_{13}^{(1)}$	$b_{14}^{(1)}$	$b_{15}^{(1)}$	$b_{16}^{(1)}$	$b_{ij}^{(1)} = \frac{a_{ij}}{a_{11}}, (j \geq 2)$	2
	$a_{22}^{(1)}$	$a_{23}^{(1)}$	$a_{24}^{(1)}$	$a_{25}^{(1)}$	$a_{26}^{(1)}$	$a_{ij}^{(1)} = a_{ij} - a_{i1} \cdot b_{1j}^{(1)}$	
	$a_{32}^{(1)}$	$a_{33}^{(1)}$	$a_{34}^{(1)}$	$a_{35}^{(1)}$	$a_{36}^{(1)}$	$a_{24}^{(1)} = a_{24} - a_{21} \cdot b_{14}^{(1)}$	1
	$a_{42}^{(1)}$	$a_{43}^{(1)}$	$a_{44}^{(1)}$	$a_{45}^{(1)}$	$a_{46}^{(1)}$	$(i, j \geq 2)$	A_1
	1	$b_{23}^{(2)}$	$b_{24}^{(2)}$	$b_{25}^{(2)}$	$b_{26}^{(2)}$	$b_{2j}^{(2)} = \frac{a_{2j}^{(1)}}{a_{22}^{(1)}}$ $(j \geq 3)$	2
		$a_{33}^{(2)}$	$a_{34}^{(2)}$	$a_{35}^{(2)}$	$a_{36}^{(2)}$	$a_{ij}^{(2)} = a_{ij}^{(1)} - a_{i2}^{(1)} \cdot b_{2j}^{(2)}$	1

		$a_{43}^{(2)}$	$a_{44}^{(2)}$	$a_{45}^{(2)}$	$a_{46}^{(2)}$	$a_{43}^{(2)} = a_{43}^{(1)} - a_{42}^{(1)} \cdot b_{23}^{(2)}$	A_2
		1	$b_{34}^{(3)}$	$b_{35}^{(3)}$	$b_{36}^{(3)}$	$b_{3j} = \frac{a_{3j}^{(2)}}{a_{33}^{(2)}} (j \geq 4)$	2
			$a_{44}^{(3)}$	$a_{45}^{(3)}$	$a_{46}^{(3)}$	$a_{ij}^{(3)} = a_{ij}^{(2)} - a_{i3}^{2} \cdot b_{3j}^{(3)}$	1
			1	$b_{45}^{(4)}$	$b_{46}^{(4)}$	$b_{4j}^{(4)} = \frac{a_{4j}^{(3)}}{a_{44}^{(3)}} (j \geq 5)$	A_3 2
			1	x_4	$\bar{x}_4$	$x_4 = b_{45}^{(4)}$	
		1		x_3	$\bar{x}_3$	$x_3 = b_{35}^{(3)} - b_{34}^{(3)} \cdot x_4$	
	1			x_2	$\bar{x}_2$	$x_2 = b_{25}^{(2)} - b_{24}^{(2)} \cdot x_4 - b_{23}^{(2)} \cdot x_3$	B
1				x_1	$\bar{x}_1$	$x_1 = b_{15}^{(1)} - b_{14}^{(1)} \cdot x_4 - b_{13}^{(1)} \cdot x_3 - b_{12}^{(1)} \cdot x_2$	

Namunaviy misol

Quyidagi tenglamalar sistemasi Gauss usulida yechilsin:

$$\begin{cases} 3{,}4x_1 + 2x_2 - 5{,}6x_3 + 4{,}8x_4 = 6{,}21 \\ 2{,}4x_1 - 3{,}7x_2 + 8{,}5x_3 - 5{,}9x_4 = 5{,}73 \\ 9{,}3x_1 + 8{,}9x_2 - 3{,}3x_3 - 0{,}6x_4 = 2{,}5 \\ 0{,}7x_1 - 7{,}4x_2 + 4{,}1x_3 + 5{,}8x_4 = 3{,}4 \end{cases}$$

Yechish: Sxemaning A bo'limining 1-qismiga noma'lumlar oldidagi koeffitsientlarni, ozod hadlarni va kontrol yig'indini yozamiz. 1-satrning barcha elementlarini 3,4 ga bo'lib, A bo'limning ikkinchi qismini to'ldiramiz.

A – bo'limning 1-qismini to'ldirish uchun uning elementlarini quyidagicha topamiz: A bo'limning 1-qismidagi 1-satrda turgan ixtiyoriy elementidan, shu element turgan satrning birinchi elementi bilan, shu element turgan ustunning oXirgi elementi ko'paytmasini ayirib, A bo'limning 1-qismidagi mos o'rniga yozamiz.

$$a_{22}^{(1)} = a_{22} - a_{21} \cdot b_{13}^{(1)} = -3{,}7 - 2{,}4 \cdot 0{,}588 = -5{,}1112$$
$$a_{23}^{(1)} = a_{23} - a_{21} \cdot b_{13}^{(1)} = 8{,}5 - 2{,}4(-1{,}647) = 12{,}4528$$
$$a_{34}^{(1)} = a_{34} - a_{31} \cdot b_{14}^{(1)} = -0{,}6 - 9{,}3 \cdot 1{,}412 = -13{,}7316$$

A_1 bo'limning 2-qismini topish uchun 1-qismning 1-satrining barcha elementlarining – 5,1112 ga bo'lamiz.

A_2, A_3 – bo'limlar ham A_1-bo'lim to'ldirilganidek to'ldiriladi. Noma'lumlarni topish uchun, birinchi koeffisienti 1 ga teng bo'lgan satrlardan foydalanamiz. $X_4 – A_3$ bo'limning oxirgi elementidir:

$a_4 = b_{45}^{(4)} = -1{,}6564$ x_1, x_2, x_3 larni quyidagicha hisoblaymiz:

$$x_3 = b_{35}^{(3)} - b_{34}^{(3)} x_4 = -0{,}666 + 0{,}9799(-1{,}6564) = -2{,}2894,$$

$$x_2 = b_{25}^{(2)} - b_{24}^{(2)} x_4 - b_{23}^{(2)} x_3 = -0{,}2637 - 1{,}8173 \cdot (-1{,}6564) + 2{,}4364 \cdot (-2{,}2894) = -2{,}8314$$

$$x_1 = b_{15}^{(1)} - b_{14}^{(1)} x_4 - b_{13}^{(1)} x_3 - b_{12}^{(1)} x_2 = 1{,}826 - 1{,}412 \cdot (-1{,}6564) - 1{,}647 \cdot (-2{,}2894) - 0{,}588 \cdot (-2{,}8314) = 2{,}0591$$

Endi $x_1 = 2{,}0591$, $x_2 = -2{,}8314$, $x_3 = -2{,}2894$, $x_4 = -1{,}6564$ quyidagilarni sxemaga yozamiz. Noma'lumlarning topilgan bu qiymatlarini berilgan sistemaga quyib, tekshiramiz.

Javob: $x_1 = 2{,}0591$, $x_2 = -2{,}8314$, $x_3 = -2{,}2894$, $x_4 = -1{,}6564$

x_1	x_2	x_3	x_4	*Ozod Xadlar*	*Yigindi* $\sum_{k=1}^{5} x_k$	*Bo'limlar*
3,4	2	-5,6	4,8	6,21	10,81	A
2,4	-3,7	8,5	-5,9	5,73	7,63	
9,3	8,9	-3,3	-0,6	2,5	16,8	
0,7	-7,4	4,1	5,8	3,4	3,4	
1	0,588	-1,647	1,412	1,826	3,179	A_1
	-5,1112	12,4528	-9,2888	1,3476	-0,5996	
	3,4316	12,0171	-13,731	-14,4818	-12,7647	
	-7,8116	5,2529	4,8116	2,1218	4,3747	
	1	-2,4364	1,8173	-0,2637	0,1173	A_2
		20,3779	-19,9678	-13,5769	-13,1672	
		-13,7793	19,0076	0,0619	5,2910	
		1	-0,9799	-0,6663	-0,6462	A_3
			5,5053	-9,1192	-3,6139	
			1	-1,6564	-0,6564	
			1	-1,6564	-1,6564	
		1		-2,2894	-1,1894	
	1			-2,8314	-1,8314	
1				2,0591	3,0591	

1-misol. Quyidagi tenglamalar sistemasi echilsin.

$$\begin{cases} 2x_1 + x_2 - x_3 = 1 \\ 3x_1 + 2x_2 - 2x_3 = 1 \\ x_1 - x_2 + 2x_3 = 5 \end{cases}$$

Yechish. Birinchi tenglamaning barcha hadlarini $a_{11}=2$ ga bo'lib,

$$\begin{cases} x_1 + 0{,}5x_2 - 0{,}5x_3 = 0{,}5 \\ 3x_1 + 2x_2 - 2x_3 = 1 \\ x_1 - x_2 + 2x_3 = 5 \end{cases}$$

sistemani hosil qilamiz. Birinchi tenglamani 3ga ko'paytirib ikkinchi tenglamadan, so'ngra uchinchi tenglamadan ayirsak quyidagi sistemani hosil qilamiz:

$$\begin{cases} x_1 + 0{,}5x_2 - 0{,}5x_3 = 0{,}5 \\ 0{,}5x_2 - 0{,}5x_3 = -0{,}5 \\ -1{,}5x_2 + 2{,}5x_3 = 4{,}5 \end{cases}$$

Ikkinchi tenglamani 0.5 ga bo'lib, so'ngra uni -1.5 ga ko'paytirib, uni uchinchi tenglamadan ayiramiz.

Natijada $\begin{cases} x_1 + 0{,}5x_2 - 0{,}5x_3 = 0{,}5 \\ x_2 - x_3 = -1 \\ x_3 = 3 \end{cases}$ hosil bo'ladi. Bundan ketma-ket $x_3=3$, $x_2=-1+3=2$, $x_1=0.5-0.5x_2+0.5x_3=1$ larni topamiz. Shunday qilib, berilgan sistemani echimi $x_1=1$, $x_2=2$, $x_3=3$ dan iborat ekan.

2-misol. $\begin{cases} x_1 + 2x_2 + 4x_3 - x_4 + 3x_5 = 7 \\ 2x_1 + x_3 + x_5 = 4 \\ x_2 + 2x_4 - x_5 = 6 \end{cases}$ sistema echilsin.

Bu sistemada uchta tenglama beshta nomalum bo'lgandan, x_4 va x_5 larni o'ng tomonga olib o'tamiz.

$$\begin{cases} x_1 + 2x_2 + 4x_3 = 7 + x_4 - 3x_5 \\ 2x_1 + x_3 = 4 - x_5 \\ x_2 = 6 - 2x_4 + x_5 \end{cases}$$

Misol uchun, $x_4=2$, $x_5=1$ qiymatlarni qo'ysak

$$\begin{cases} x_1 + 2x_2 + 4x_3 = 6 \\ 2x_1 + x_3 = 3 \\ x_2 = 3 \end{cases}$$

sistema hosil bo'ladi. $x_2=3$ ekanini e'tiborga olib,

$$\begin{cases} x_1 + 4x_3 = 0 \\ 2x_1 + x_3 = 3 \end{cases}$$

sistemaga ega bo'lamiz. Birinchi tenglamani ikkiga ko'paytirib, undan ikkinchi tenglamani ayirsak

$$\begin{cases} x_1 + 4x_3 = 0 \\ 7x_3 = -3 \end{cases}$$

hosil bo'ladi. Bundan $x_3 = -3/7$, $x_2 = 3$, $x_1 = 12/7$.

3-misol. $\begin{cases} 2x_1 + x_2 - 5x_3 + x_4 = 8 \\ x_1 - 3x_2 - 6x_4 = 9 \\ 2x_2 - x_3 + 2x_4 = -5 \\ x_1 + 4x_2 - 7x_3 + 6x_4 = 0 \end{cases}$ tenglamalar sistemasi echilsin.

Yechish. Berilgan sistemani Kramer usulida echaylik.

$$\Delta = \begin{vmatrix} 2 & 1 & -5 & 1 \\ 1 & -3 & 0 & -6 \\ 0 & 2 & -1 & 2 \\ 1 & 4 & -7 & 6 \end{vmatrix} = 2\begin{vmatrix} -3 & 0 & -6 \\ 2 & -1 & 2 \\ 4 & -7 & 6 \end{vmatrix} - \begin{vmatrix} 1 & -5 & 1 \\ 2 & -1 & 2 \\ 4 & -7 & 6 \end{vmatrix} + 0\begin{vmatrix} 1 & -5 & 1 \\ -3 & 0 & -6 \\ 4 & -7 & 6 \end{vmatrix} - \begin{vmatrix} 1 & -5 & 1 \\ -3 & 0 & -6 \\ 2 & -1 & 2 \end{vmatrix} = 27$$

$\Delta x_1 = 81$, $\Delta x_2 = -108$, $\Delta x_3 = -27$, $\Delta x_4 = 27$.

Demak, sistema yagona echimga ega, chunki $\Delta \neq 0$. Bu echim esa

$x_1 = \Delta x_1/\Delta = 3$, $x_2 = \Delta x_2/\Delta = -4$, $x_3 = \Delta x_3/\Delta = -1$, $x_4 = \Delta x_4/\Delta = 1$. bo'ladi.

(1) tenglamalar sistemasi bir jinsli, ya'ni $b_1 = b_2 = ... = b_n = 0$ bo'lgan holni ko'ramiz.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = 0 \end{cases} \qquad (2)$$

(2) tenglamalar sistemasi bir jinsli, chiziqli tenglamalar sistemasi deyiladi .

Ishonch hosil qilish mumkinki, $x_1 = x_2 = ... = x_n = 0$ (2) ning echimi bo'ladi va bu echimni trivial echim deb ataladi. Agar (2) bir jinsli sistemaning asosiy determinanti Δ noldan farqli bo'lsa, bu sistema fakat trivial echimga ega bo'ladi.

chunki bu holda $\Delta x_1=\Delta x_2 = ... =\Delta x_n= 0$ bolgani uchun Kramer formulasiga asosan $x_1=x_2=...=x_n =0$ bo'ladi .

Demak (2) sistemani notrivial echimi mavjud bo'lishi uchun $\Delta=0$ bo'lishi zarur.

4-misol. $\begin{cases} -x_1 + x_2 = 0 \\ x_1 - x_2 = 0 \end{cases}$ tenglamalar sistemasi echilsin.

Yechish. $x_1=x_2=0$ trivial echim ekani ravshan. $\Delta = \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} = 1-1 = 0,$

bundan ko'rinadiki, sistemaning notrivial echimi bo'lishi mumkin. Hakikatan ham $x_1=t$, $x_2=t$ (t-ixtiyoriy haqiqiy son) sistemaning notrivial echimi bo'ladi.

Bu sistemada x_4 va x_5 nomalumlarga boshqa qiymatlar berib, yangi echim xhosil qilish mumkin ekanini, boshqacha aytganda n>m bo'lganda echim yagona bo'lmay cheksiz ko'p bo'lishini eslab o'tamiz.

Etibor qiladigan bo'lsak, sistemani yechishning Gauss usulida, sistemani birgalikda ekanini oldindan aniqlab olinmaydi. Tenglamalar sistemasini birgalikda bo'lish-bo'lmasligini, uni echmasdan turib aniqlash usuli bilan tanishamiz.

(1) tenglamalar sistemani koefitsentlaridan tuzilgan m*n tartibli

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

hamda m·(n+1) - tartibli kengaytirilgan

$$A' = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n}b_1 \\ a_{21} & a_{22} & \dots & a_{2n}b_2 \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn}b_m \end{pmatrix}$$

matritsalarni tuzib olamiz.

3.6.1. Kroneker-Kappelli teoremasi

Teorema. (Kroneker-Kappelli teoremasi)

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{cases}$$ tenglamalar sistemasi birgalikda bo'lishi uchun A va A' matritsalarning ranglari teng bo'lishi, ya'ni rang A= rang A' bo'lish zarur va etarli.

Keltirilgan teoremadan quyidagi xulosalar kelib chiqadi:

1. Agar rang A' > rang A bo'lsa (1) sistema echimga ega bo'lmaydi.
2. Agar rang A = rang A' = k bo'lsa, sistema echimga ega bo'lib,

a) k<n bo'lganda, tenglama cheksiz ko'p echimga ega bo'ladi;

b) k=n bo'lsa, sistema yagona echimgaega bo'ladi.

3-misol. $\begin{cases} 7x_1 + 3x_2 = 2 \\ x_1 - 2x_2 = 2 \\ 4x_1 + 9x_2 = 11 \end{cases}$ tenglamalar sistemasi esilsin.

Yechish. Bu erda n=2, m=3 ya'ni m>n.

$$\mathrm{A} = \begin{pmatrix} 7 & 3 \\ 1 & -2 \\ 4 & 9 \end{pmatrix}, \quad A' = \begin{pmatrix} 7 & 3 & 2 \\ 1 & -2 & -3 \\ 4 & 9 & 11 \end{pmatrix}$$

rang A =2 chunki $\begin{vmatrix} 7 & 3 \\ 1 & -2 \end{vmatrix} = -17 \neq 0$ $\begin{vmatrix} 7 & 3 & 2 \\ 1 & -2 & -3 \\ 4 & 9 & 11 \end{vmatrix} = -172 + 172 = 0$ bo'lishini etiborga olsak rang A'=2, demak bu sistemani echimi mavjud. Berilgan sistemani birinchi ikki tenglamasini birgalikda echsak $x_1 = -{}^{5}/_{17}$; $x_2 = {}^{23}/_{17}$, kilib chiqadi. Bu sonlar uchinchi tenglamani ham qanoatlantiradi.

$$4x_1 + 9x_2 = 4 \cdot (-5/17) + 9 \cdot 23/17 = 11.$$

Demak, $(-{}^{5}/_{17};\ {}^{23}/_{17})$ sistemaning echimi bo'ladi.

IV Bob

TEKISLIKDA ANALITIK GEOMETRIYA ELEMENTLARI

4.1-§ Chiziq tenglamasi

Faraz qilaylik, x ва y o'zgaruvchi miqdorlar bo'lsin. Agar F(x,y)=0 tenglik x va y ning har qanday qiymatlarida to'g'ri bo'lsa y tenglama deyiladi. Masalan $5x+2y-4=0$, $x^2+y^2-16=0$, $\sin x+\cos y-1=0$ va hokazo.

Agarda F(x,y)=0 tenglik har qanday x va y lar uchun to'g'ri bo'lsa, u ayniyat deyiladi. Masalan: $(x-y)^2-x^2+2xy-y^2=0$, $(x+y)(x-y)-x^2+y^2=0$ va h.k.

Tenglamalar x va y dan boshqa yana a,b,c kabi o'zgarmaslarga bo'g'liq bo'lishi ham mumkin. Bu o'zgarmaslar parametrlar deyiladi. Masalan: $ax+by+c=0$

Agarda F(x,y)=0 tenglama bo'lsa va x biror aniq son qiymat qabul qilsa, u vaqtda y endi har qanday qiymat qabul qila olmaydi. Bunday holda y faqat tenglamadan kelib chiqadigan bir qiymatni majburiy ravishda qabul qiladi. Shu ham F(x,y)=0 tenglik x va y o'rtasida biror funksional bog'liqlik o'rnatadi.

Chiziq tenglamasi nima? Tanlangan koordinata tizimida berilgan chiziq tenglamasi deb shunday ikkita o`zgaruvchiga bog`lik F(x,y)q0 tenglamaga aytiladiki, bu chiziqda yotuvchi xar qanday nuqta koordinatalar tenglamani qanoatlantiradi va bu chiziqda yotmovchi hech bir nuqta koordinatalari esa tenglamani kanoatlantirmaydi. Shuning uchun chiziqning o`zi o`rniga uning tenglamasini tadqiq qiladilar. Chiziq funktsiya grafigi deyiladi.

Misol. а) x-y=0 ва x+y=0 tenglamalar mos ravishda I,III ва II,IV choraklar bissektrisalardir;

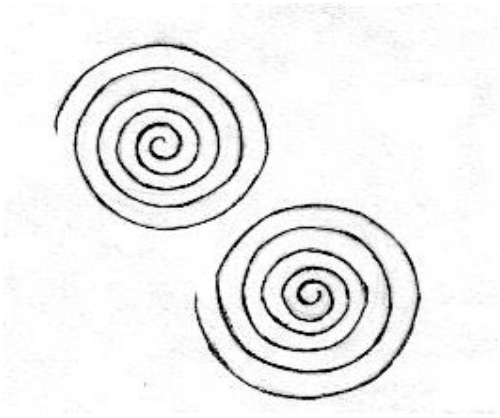
б) $x^2-y^2=0$ - koordinata burchaklari bissektrisalari;

в) $x^2+y^2=0$ - faqat O(o;o) nuqtani aniqlaydi, xolos.

г) $x^2+y^2+1=0$ - hech bir geometrik obrazni aniqlamaydi;

д) $\rho=a\cdot\cos 0$, a>o ва p, - qutb koordinatalari.

е) $\rho=a\cdot\theta$ - Arximed spirali a>o da soat strelkasiga teskari va a<0 da esa soat strelkasi yo`nalishi bo`yicha, spirallardir.



1 - chizma

4.1.1. Bronchi tartibli chiziqlar

To’g’ri chiziq birinchi tartibli chiziq sifatida. To’g’ri chiziqning chala tenglamalari

Ax+By+C=0 (1) => **y**= $-\frac{A}{B}\cdot x-\frac{C}{B}$= kx+ b (2) to’g’ri chiziq tenglamasi, demak (1) ham to’g’ri chiziq ekan. (1) - to’g’ri chiziqning umumiy (2) – burchak koefisientli tenglamasi deyiladi.

To’g’ri chiziqning umumiy tenglamasidan, koeffitsientlar va ozod hadni nolga tenglash orqali to’g’ri chiziqning chala tenglamalarini hosil qilamiz.

a) Agar A=0 bo’lsa, By=-C va y=-$\frac{C}{B}$=b Oy o’qqa perpendikulyar, yoki Ox o’qqa parallel to’g’ri chiziq tenglamasi hosil bo’ladi.

б) Agar B=0 bo’lsa, Ax+C=0 va x=-$\frac{A}{B}$=a Ox o’qqa perpendikulyar yoki Oy o’qqa parallel to’g’ri chiziq tenglamasi hosil bo’ladi.

в) Agar C=0 bo’lsa Ax+By=0 - koordinata boshidan o’tuvchi to’g’ri chiziq tenglamasi y=-$\frac{A}{B}$x hosil bo’ladi.

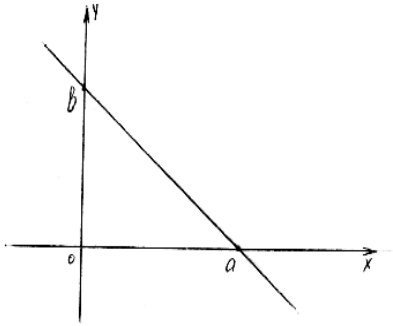
g) Agar A=C= 0 bo’lsa, By=0 yoki y=0 Ox absisa o’qi tenglamasi hosil bo’ladi.

d) Agar B=C= 0 bo’lsa, Ax=0 yoki x=0 Oyordinata o’qi tenglamasi hosil bo’ladi.

4.1.2. To’g’ri chiziqning kesmalar bo’yicha tenglamasi

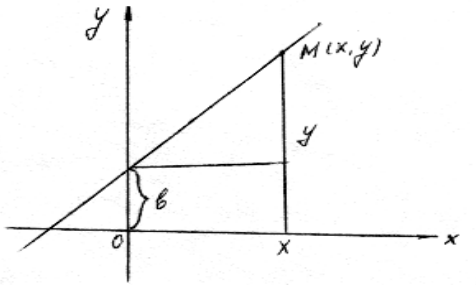
To’g’ri chiziqning kesmalar bo’yicha tenglamasi uning umumiy tenglamsini shakl almashtirish yordamida hosil qilinadi

Ax+By+C=O $\Rightarrow \frac{Ax}{-C}+\frac{By}{-C}=1\Rightarrow$ $\frac{x}{\frac{-C}{A}}+\frac{y}{\frac{-C}{B}}=1\Rightarrow \frac{x}{a}+\frac{y}{b}=1$ - to'g'ri chiziqning kesmalar bo'yicha tenglamasidir.



2 – chizma

Berilgan M(x,y) nuqtadan o'tib Ox o'qi bilan musbat yo'nalishi bilan α-burchak tashkil qiluvchi to'g'ri chiziqning burchagi tangensi tgα- bo'lsa, to`g`ri chiziqning Ox o'qi hosil qilgan yani burchak koeffisenti **k**=tg(α+nπ) ga teng bo'ladi, bunda n- butun son.

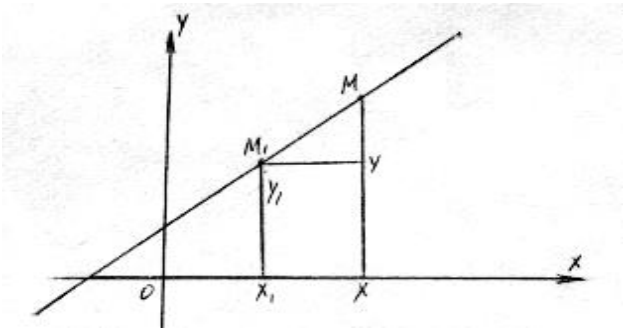


3 – chizma

ζ_2

$M_1(x_1; y_1)$ va $M_2(x_2; y_2)$ nuqtalar berilgan bo`lsa, u holda to`g'ri chiziqning burchak koeffisienti $k=\frac{y_2-y_1}{x_2-x_1}$ ga teng bo`ladi.

To`ri chiziqning burchak koeffisientli tenglamasi esa $\frac{y-b}{x}=k$ ko'rinishda, yoki **y**=kx+b ko'rinishda bo`ladi.



3 - chizma

3. Endi berilgan $M_0(x_0, y_0)$ nuqtadan o'tib, berilgan k burchak koeffitsientli to'g'ri chiziq tenglamasini yozaylik. To'g'ri chiziq ordinata o'qiga parallel bo'lmasin. Uning tenglamasi **y**=kx+b ko'rinishda bo'lib, $M_0(x_0,y_0)$ nuqtadan o'tadi. **y**=kx+b dan $y_0 = kx_0 + b$ ni ayirsak $y-y_0=k(x-x_0)$ kelib chiqadi.

4. Tto'g'ri chiziqda $M_1(x_1,y_1)$, $M_2(x_2,y_2)$ nuqtalar orqali berilgan bo'lsin va to'g'ri chiziq (Oy) ka parallel bo'lmasin. Uning burchak koeffitsienti

$$k = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_2 - x_1 \neq 0$$

Topilgan burchak koeffitsient qiymatini $y-y_0=k(x-x_0)$ ga qo'ysak,

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

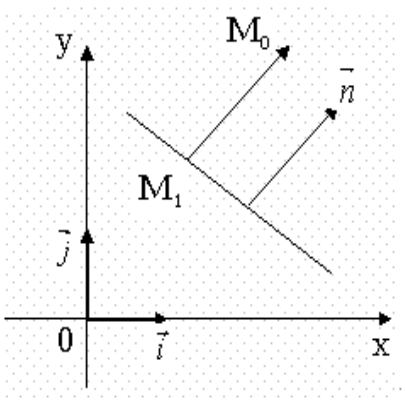
berigan ikki nuqtadan o`tuvchi to`g`ri chiziq tenglamasi kelib chiqadi.

4.1.3. To'g'ri chiziqning normal tenglamasi. Nuqtadan to'g'ri chiziqgacha bo'lgan masofa

1. Faraz qilaylik OP=p α-normalning qutb burchagi. OM=ρ va θ-lar M(ρ;θ) - qutb koordinatalari, M(x,y) -nuqtaning Dekart koordinatalari bo'lsin.

U holda $\Pi rOM=\rho\cos\varphi=\rho\cos(\theta-\theta_1)=\rho\cos\theta\cdot\cos\alpha+\rho\sin\theta\cdot\sin\alpha=p$. (*) $\rho\cos\theta=x$, $\rho\sin\theta=y$ bo'lgani uchun (*) $x\cos\alpha+y\sin\alpha-\rho=0$ – to'g'ri chiziqning normal tenglamasini hosil qilamiz.

2. $B=\{0,\vec{i},\vec{j}\}$ to'g'ri burchakli Dekart koordinatalar sistemasi bo'lsin. $\vec{u}=\{-B,A\}$ vektor $l: Ax+By+C=0$ to'g'ri chiziqning yo'naltiruvchi vektori bo'lsa, $\vec{n}=\{A,B\}$ vektor l to'g'ri chiziqning normal vektori deyiladi. $\vec{u}$ va $\vec{n}$ vektorlar o'zaro perpendikulyar bo'lib, ularning skalyar ko'paytmasi nolga teng bo'ladi, ya'ni $(\vec{u}\cdot\vec{n})=0.$



3. Agar to'g'ri chiziqning $A_0x+B_0y+C_0=0$ (1) tenglamasida $|\vec{n}|=\sqrt{A_0{}^2+B_0{}^2}=1$ shart bajarilsa, u holda (1) ni to'g'ri chiziqning normal tenglamasi deyiladi.

To’g’ri chiziq umumiy tenglamasini normal ko’rinishga keltirish uchun tenglamaning chap tomonini $\left|\vec{n}\right|$ ga bo’lamiz: $N=\frac{1}{\sqrt{A^2+B^2}}$ (2)

Bu kattalikni normallashtiruvchi ko’paytuvchi deyiladi. Tekislikni umumiy tenglamasini normallashtiruvchi ko’paytuvchiga ko’paytirib uni normal tenglamasi $\frac{A}{\sqrt{A^2+B^2}}x+\frac{B}{\sqrt{A^2+B^2}}y+\frac{C}{\sqrt{A^2+B^2}}=0$ yoki $x\cos\alpha+y\sin\alpha-\rho=0$ ni hosil qilamiz.

Ushbu tenglamalarning 1-sidagi tenglama koeffitsientlarini A_0, B_0 va C_0 orqali belgilab, (1) tenglamani hosil qilamiz. Haqiqatdan ham,

$$\left(\frac{A}{\sqrt{A^2+B^2}}\right)^2+\left(\frac{B}{\sqrt{A^2+B^2}}\right)^2=A_0^{\,2}+B_0^{\,2}=1.$$ yoki $\sin^2\alpha+\cos^2\alpha=1$

4. Dekart koordinatalar sistemasida $M_0(x_0, y_0)$ nuqta va l to’g’ri chiziq umumiy tenglamasi $l: Ax+By+C=0$ orqali berilgan bo’lsin. M_0 nuqtadan l to’g’ri chiziqqa perpendikulyar o’tkazamiz.

Perpendikulyarning l dagi asosini $M_1(x_1, y_1)$ bilan belgilaymiz. $\overrightarrow{M_1M_0}$ vektorning uzunligini M_0 nuqtadan l to’g’ri chiziqqacha bo’lgan masofa deyiladi va $\rho(M_0,l)$ ko’rinishida belgilanadi.

$M_0\in l$ uchun $\rho(M_0,l)=0.$ $M_0\notin l$ bo’lsin. $\vec{n}=\{A,B\}\perp l$ normal vektor bo’lgani uchun $\overrightarrow{M_0M_1}$ va $\vec{n}$ vektorlar o’zaro kollinear bo’ladi. $\overrightarrow{M_0M_1}$ va $\vec{n}$ vektorlar bir hil yoki qarama-qarshi yo’nalgan bo’lishi mumkin, ya’ni $\cos(\overrightarrow{M_1M_0},\vec{n})=\pm1.$

$$(\overrightarrow{M_1M_0},\vec{n})=\left|\overrightarrow{M_1M_0}\right|\cdot\left|\vec{n}\right|\cdot\cos(\overrightarrow{M_1M_0}\,\hat{}\,\vec{n})=\pm\rho(M_0,l)\cdot\left|\vec{n}\right|.$$

Agar $\overrightarrow{M_1M_0}=\{x-x_0,y_0-y_1\},$ $\vec{n}=\{A,B\}$ bo’lsa,

$$\rho(M_0,l)=\frac{\left|\left(\overrightarrow{M_1M_0},\vec{n}\right)\right|}{\left|\vec{n}\right|}.=\frac{\left|Ax_0+By_0+C\right|}{\sqrt{A^2+B^2}}.$$

Bunda $Ax_1+by_1+C=0\Rightarrow C=-(Ax_1+By_1)$ dan foydalandik.

<u>Xulosa</u>: $M_0(x_0,u_0)$ nuqtadan $l: Ax+By+C=0$ to’g’ri chiziqgacha bo’lgan

masofani hisoblash formulasi $\rho(M_0,l)=\dfrac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$ (4)

ko'rinishga ega. Masalan: $l:3x-4y-10=0$ va $M_0(5,-5)$ berilgan. $\rho(M_0,l)$ masofani aniqlalaylik.

$$\rho(M_0,l)=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}=\frac{|3\cdot 5-4\cdot(-5)+10|}{\sqrt{3^2+4^2}}=\frac{25}{5}=5.$$

4.1.4 Ikki to'g'ri chiziq orasidagi burchak.
To'g'ri chiziqlarning parallellik va perpendikulyarlik shartlari

Tekislikdagi dekart koordinatalar sistemasida l_1 va l_2 to'g'ri chiziqlar

$$\begin{aligned} &l_1;A_1x+B_1y+C_1=0\\ &l_2;A_2x+B_2y+C_2=0 \end{aligned} \qquad (1)$$

tenglamalar bilan berilgan bo'lsin.

Ta'rif: Ikki to'g'ri chiziq orasidagi burchak deb bu to'g'ri chiziqlarning yo'naltiruvchi vektorlari orasidagi burchakka aytiladi. $\overrightarrow{u_1}=\{-B_1,A_1\}$ l_1 to'g'ri chiziqning, $\overrightarrow{u_2}=\{-B_2,A_2\}$ l_2 to'g'ri chiziqning yo'naltiruvchi vektoridir. U holda ularning skalyar ko'patmasi $\left(\overrightarrow{u_1},\overrightarrow{u_2}\right)=\left|\overrightarrow{u_1}\right|\cdot\left|\overrightarrow{u_2}\right|\cos(l_1\hat{}\,l_2)\Rightarrow$ u holda ular orasidagi burchak kosinusi

$$\cos(l_1\hat{}\,l_2)=\frac{\left(\overrightarrow{u_1},\overrightarrow{u_2}\right)}{\left|\overrightarrow{u_1}\right|\cdot\left|\overrightarrow{u_2}\right|}=\frac{A_1A_2+B_1B_2}{\sqrt{A_1^{\,2}+B_1^{\,2}}\sqrt{A_2^{\,2}+B_2^{\,2}}} \qquad (2)$$ kabi aniqlanadi.

Agar $\overrightarrow{u_1}\perp\overrightarrow{u_2}\Leftrightarrow l_1\perp l_2$ U holda to'g'ri chiziqlar o'zaro perpendikulyar bo'ladi, yani

$$l_1\perp l_2\Leftrightarrow A_1A_2+B_1B_2=0 \qquad (3)$$

(3) tenglik l_1 va l_2 to'g'ri chiziqlar perpendikulyarligining zaruriy va etarli shartidir. l_1 va l_2 to'g'ri chiziqlar burchak koeffitsientli tenglamalari bilan berilgan bo'lsin. $l_1:y=k_1x+b_1;\ \ l_2:y=k_2+b_2$ (4)

Bu to’g’ri chiziqlar orasidagi burchakni hisoblash formulasini keltirib chiqaramiz. Ma’lumki, l to’g’ri chiziqning yo’naltiruvchi vektori $\vec{a}\{a_1, a_2\}$ Ox o’qining musbat yo’nalishi bo’yicha α va α’ burchak tashkil etadi. α burchak o’tkir bo’lsa, α’ o’tmas bo’lib, $\alpha' = \alpha + \pi$ tenglik o’rinli bo’ladi.

$$\pi r_{\vec{i}} = a_1, \quad \pi r_{\vec{j}} = a_2 \text{ bo’lib, } a_1 = \left|\overrightarrow{u}\right| \cos\alpha, \quad a_2 = \left|\overrightarrow{u}\right| \sin\alpha \qquad (5)$$

$$k = \frac{a_2}{a_1} = tg\alpha = tg\alpha'. \qquad (6)$$

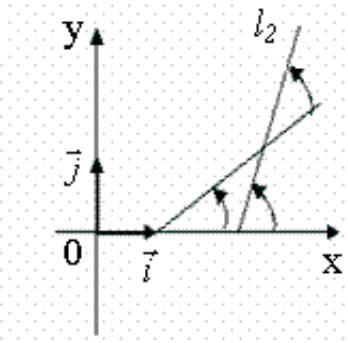
Demak, l to’g’ri chiziqning burchak koeffitsienti k l ning Ox o’qning musbat yo’nalishi bilan tashkil etgan α va α’ burchaklardan har birining tangensiga teng.

$\left(\overrightarrow{u_1} \hat{} \vec{l}\right)$ - α ni l ning (Ox) o’qqa og’ish burchagi deyiladi.

l_1 va l_2 to’g’ri chiziqlarning (Ox) o’qqa og’ish burchaklari φ_1 va φ_2 bo’lsin (1-chizma). U holda

$$k_1 = tg\varphi_1, \; k_2 = tg\,\varphi_2 \text{ va } \left(l_1 \hat{} l_2\right) = \varphi_2 - \varphi_1. \qquad (7)$$

$$tg(l_1 \hat{} l_2) = tg(\varphi_2 - \varphi_1) = \frac{tg\varphi_2 - tg\varphi_1}{1 + tg\varphi_1\, tg\varphi_2} \Rightarrow tg(l_1 \hat{} l_2) = \frac{k_2 - k_1}{1 + k_1 k_2} \qquad (8)$$

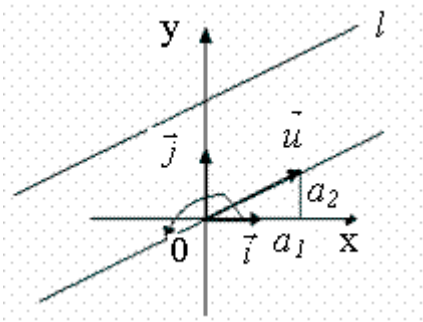


1-chizma

(8) dan l_1 va l_2 to’g’ri chiziqlarning parallellik va perpendikulyarlik shartlari kelib chiqadi:

$$l_1 \,//\, l_2 \Leftrightarrow k_1 = k_2 \qquad (9)$$

$$l_1 \perp l_2 \Leftrightarrow k_1 \cdot k_2 = -1 \qquad (10)$$



2-chizma

Misol-1: $l_1 : y = 2x - 1,\ \ l_2 : y = -x + 1$ to'g'ri chiziqlar orasidagi burchak aniqlansin.

Yechish: $tg\left(l_1 \hat{} l_2\right) = \dfrac{-1-2}{1-2} = 3.$

Misol-2: $\begin{cases} x = a_1 t + b_1 \\ y = a_2 t + b_2 \end{cases}$ va $\begin{cases} x = c_1 t + d_1 \\ y = c_2 t + d_2 \end{cases}$ parametrik ko'rinishda berilgan to'g'ri chiziqlar orasidagi burchakni aniqlang.

Yechish: $k_1 = \dfrac{a_2}{a_1},\ k_2 = \dfrac{c_2}{c_1};\ tg\varphi = \dfrac{k_2 - k_1}{1 + k_1 \cdot k_2} = \dfrac{a_1 c_2 - a_2 c_1}{a_1 c_1 + a_2 c_2}.$

Mavzu bo`yicha takrorlash savollari

1. Tenglama va ayniyat nima?
2. Parametr nima?
3. Chiziq tenglamasi nima?
4. To`g`ri chiziqning burchak koeffitsienti qanday aniqlanadi?
5. To`g`ri chiziqning burchak koeffitsienti tenglamasini yozing?
6. Berilgan bitta va berilgan ikki nuqtadan o`tuvchi to`g`ri chiziq tenglamalarini yozing?
7. To`g`ri chiziqning parallellik va perpendikulyarlik shartlarini yozing?
8. To`g`ri chiziqning umumiy tenglamasi va chala tenglamalarini yozing?
9. To`g`ri chiziqning kesmalari bo`yicha tenglamasini yozing?
10. To`g`ri chiziqning normal tenglamasini yozing?
11. Nuqtadan to`g`ri chiziqqacha masofa qanday hisoblanadi?

Bobga oid masalalar echish namunalari va mustaqil yechish uchun topshiriqlar

Misol 1. $M_1(3;1)$ $M_2(2;3)$ $M_3(6;3)$ $M_4(-3;3)$ $M_5(3;-1)$ $M_6(-2;1)$ nuqtalarning qaysilari $2x - 3y - 3 = 0$ to'g'ri chiziqda yotadi, qaysilari yotmaydi?

Yechish:

$M_1(3;1);\ M_3(6;3);\ M_4(-3;3)$ nuqtalar $2x - 3y - 3 = 0$ to'g'ri chiziqda yotadi.

$M_2(2;3)$; $M_5(3;-1)$; $M_6(-2;1)$ nuqtalar $2x-3y-3=0$ to'g'ri chiziqda yotmaydi.

Misol 2. Uchburchakning yuzi $S=8$ ga teng. Uning ikki balandligi $A(1;-2)$ $B(2;3)$ nuqtalarda, uchinchi balandligi C 2x+y-2=0 to'g'ri chiziqda yotadi .C balandlikning koordinatalarini toping.

Yechish:

$$S=\pm\frac{1}{2}\begin{vmatrix} x_2-x_1 & y_2-y_1 \\ x_3-x_1 & y_3-y_1 \end{vmatrix} \quad 8=\pm\frac{1}{2}\begin{vmatrix} 1 & 5 \\ x_3-1 & y_3+2 \end{vmatrix} \Rightarrow y+2-5x+5=16$$

$$\begin{cases} y-5x=9 \\ 2x+y=2 \end{cases} \quad \begin{cases} 5x-y=-9 \\ 2x+y=2 \end{cases} \quad \begin{cases} 5x-y=-9 \\ 7x=-7 \end{cases} \quad \begin{cases} x=-1 \\ y=4 \end{cases} \quad C(-1;4)$$

№ 3. Oy o'qdan $b=5$ kesma ajratib , Ox o'q bilan 1)45° 2)135° burchaktashkil qiluvchi to'g'ri chiziqlar yasalsin.O'sha to'g'ri chiziqlarning tenglamalari yozilsin

№ 4. Oy o'qdan $b=-3$ kesma ajratib ,Ox o'q bilan 1)60° 2)120° burchak tashkil qiluvchi to'g'ri chiziqlar yasalsin . Bu to'g'ri chiziqlarning tenglamalari yozilsin .

№ 5. Koordinatalar boshidan o'tib , Ox o'q bilan 1)45° 2)60° 3)90° 4)120° 5)135° burchak tashkil qiluvchi to'g'ri chiziqlarning tenglamasi yozilsin .

№ 6. 1) $2x-5y=6;$ 2)$3x-2y=0;$ 3)$y=-3;$ 4)$\frac{x}{4}+\frac{y}{3}=1$ to'g'ri chiziqlarning har qaysisi uchun R va b parametrlar aniqlansin .

№ 7. 1) $2x-3y=6;$ 2)$3x-2y+4=0$ to'g'ri chiziqlarning tenglamalari o'qlardan ajratgan kesmalarig nisbatan yozilsin .

№ 8. A(3;5) , B(2;7) , C(-1;-3) va D(-2;-6) nuqtalar y=2x-1 to'g'ri chiziqdayotadimi, yo o'sha to'g'ri chiziqdan «yuqoriroqda» yoki «quyiroqda» joylashganmi?

№ 9. Quyidagi to'g'ri chiziqlar orasidagi burchak aniqlansin :

1) $\begin{cases} y=2x-3 \\ Y=\frac{1}{2}x+1 \end{cases}$ 2) $\begin{cases} 5x-y+7=0 \\ 2x-3y+1=0 \end{cases}$ 3) $\begin{cases} 2x+y=0 \\ y=3x-4 \end{cases}$ 4) $\begin{cases} 3x+2y=0 \\ 6x+4y+9=0 \end{cases}$ 5) $\begin{cases} \frac{x}{a}+\frac{y}{b}=1 \\ \frac{x}{b}+\frac{y}{a}=1 \end{cases}$

№ 10. 3x-2y+7=0 6x-4y-9=0 6x+4y-5=0 2x+3y-6=0 to'g'ri chiziqlardan parallel va perpendikulyar bo'lganlarini ko'rsating.

№ 11. 2x-5y-10=0 to'g'ri chiziqning koordinata o'qlari bilan kesishgan nuqtalaridan bu to'g'ri chiziqqa perpendikulyar chiqarilgan . Ularning tenglamalari yozilsin .

№ 12. A(-1;3) va B(4;-2) nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi yozilsin.

№ 13. a) A(1;7) nuqtadan y=3x-4 to'g'ri chiziqqacha bo'lgan masofa topilsin .

b) M(-2;3) nuqtadan $3x-4y+5=0$ to'g'ri chiziqqacha bo'lgan masofa topilsin.

№ 14. a) $M_0(1;7;5)$ nuqtadan $3x-y+2z-5=0$ tekislikkacha bo'lgan masofani toping. b) $M_0(2;-3;4)$ nuqtadan $2x+y-3z+4=0$ tekislikkacha bo'lgan masofani toping.

№ 15. ABC uchburchakning uchlari berilgan. Quyidagilarni toping:

a) AB tomon tenglamasini;

b) C uchidan AB tomonga tushirilgan balandlik tenglamasini;

в) A uchidan BC tomonga tushirilgan mediana tenglamasini;

г) C nuqtadan AB to'g'ri chiziqqacha bo'lgan masofani.

1. A(4;-5), B(6;9), C(-4;-1) ;
2. A(1;-3), B(-5;4), C(-2;10) ;
3. A(1;8), B(-5;-4), C(-1;-3) ;
4. A(0;4), B(5;-3), C(-6;-2) ;
5. A(6;-4), B(-8;3), C(-2;-7) ;
6. A(2;3), B(-4;-7), C(2;0) ;

№ 16. $M_1(3;1)$ $M_2(2;3)$ $M_3(6;3)$ $M_4(-3;3)$ $M_5(3;-1)$ $M_6(-2;1)$ nuqtalarning qaysilari $2x-3y-3=0$ to'g'ri chiziqda yotadi, qaysilari yotmaydi?

№ 17. P_1 P_2 P_3 P_4 va P_5 nuqtalar $3x-2y-6=0$ to'g'ri chiziqda joylashgan absissalari 4, 0, 2, -2 va -6 ga teng ordinatalarini toping.

№ 18. Q_1 Q_2 Q_3 Q_4 va Q_5 nuqtalar $3x-2y-6=0$ to'g'ri chiziqda joylashgan. Ordinatalari 1, 0, 2, -1, 3 . Absissalarini toping.

№ 19. 2x-3y-12=0 to'g'ri chiziq bilan Ox va Oy koordinatalari bilan kesishish nuqtalarini toping.1)y=0 2x=12 x=6 2)x=0 -3y=12 y=-4

№ 20. $\begin{cases}3x-4y-29=0\\2x+5y+19=0\end{cases}$ Ikki to'g'ri chiziqning kesishish nuqtasini toping.

№ 21. 4x+3y-5=0 x-3y+10=0 x-2=0 to'g'ri chiziqlar berilgan uchburchak kesishgan uchlarini toping. 1) $A(x_1;y_1)$ $B(x_2;y_2)$ $C(x_3;y_3)$ координаталари .

№ 22. Parallelogramning ikki tomon tenglamasi quyidagilar: 8x+3y+1=0 2x+y-1=0 diagonalining tenglamasi: 3x+2y+3=0. Parallelogramning uchlarining koordinatalarini toping.

№ 23. Uchburchak tomonlari $x+5y=7$ $3x-2y=4$ $7x+y=-19$ to'g'ri chiziqda yotadi . S=? toping.

№ 24. Uchburchakning yuzi $S=8$ ga teng. Uning ikki balandligi $A(1;-2)$ $B(2;3)$ nuqtalarda, uchinchi balandligi C 2x+y-2=0 to'g'ri chiziqda yotadi .C balandlikning koordinatalarini toping.

№ 25. k burchak koeffitsiyentini va b kesmani bilgan holda, Oy o'qidan o'tuvchi, to'g'ri chiziq tenglamasi tuzilsin.

1) $k=\frac{2}{3}$ $b=3$ $y=kx+b$ $y=\frac{2}{3}x+3$ $2x-3y+9=0$;
2) $k=3$ $b=0$ $y=3x$ $3x-y=0$;
3) $k=0$ $b=-2$ $y=-2$ $y+2=0$;
4) $k=-\frac{3}{4}$ $b=3$ $y=-\frac{3}{4}x+3$ $3x+4y-12=0$;
5) $k=-2$ $b=-5$ $y=-2x-y-5=0$ $2xy5=0$;
6) $k=-\frac{1}{3}$ $b=\frac{2}{3}$ $y=-\frac{1}{3}x+\frac{2}{3}$ $x+3y-2=0$;

№ 26. Har bir to'g'ri chiziq uchun Oy o'qidan o'tuvchi burchak koeffitsienti k va b kesmani toping.

1) $5x-y+3=0$ $k=5$ $b=3$
2) $2x+3y-6=0$ $k=-\frac{2}{3}$ $b=2$
3) $5x+3y+2=0$ $k=-\frac{5}{3}$ $b=-\frac{2}{3}$
4) $3x+2y=0$ $k=-\frac{3}{2}$ $b=0$
5) $y-3=0$ $k=0$ $b=3$

№ 27. 5x+3y-3=0 to'g'ri chiziq berilgan. to'g'ri chiziqda burchak koeffisiyenti k ni toping: 1) to'g'ri chiziqqa parallel bo'lgan;
2) to'g'ri chiziqqa perpendikulyar bo'lgan .

№ 28. 2x+3y+4=0 to'g'ri chiziq berilgan , $M_0(2;1)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing : 1) to'g'ri chiziqqa parallel bo'lgan ; 2) to'g'ri chiziqqa perendikulyar bo'lgan.

№ 29. Berilgan to'g'ri chiziqlarga parallel va ular o'rtasidan o'tuvchi to'g'ri chiziq tenglamalarini tuzing.

1) $3x-2y-1=0,\ 3x-2y-13=0;$ 4) $5x+7y+15=0,\ 5x+7y+3=0;$

2) $5x+y+3=0,\ 5x+y-17=0;$ 5) $3x-15y-1=0,\ x-5y-2=0;$

3) $2x+3y-6=0,\ 4x+6y+17=0;$

№ 30. Ikki nuqtadan o'tuvchi to'g'ri chiziqda burchak koeffisiyenti k ni toping :

$a)M_1(2;-5),\ M_2(3;2);\quad b)P(-3;1),\ Q(7;8);\quad c)A(5;-3),\ B(-1;6);$

№ 31. $A(5;-4),\ B(-1;3),\ C(-3;-2)$ Uchburchak uchlaridan o'tuvchi to'g'ri chiziq tenglamasi tuzilsin.

№ 32. Uchburchak tomonlarining o'rta $M_1(2;1),\ M_2(5;3)\ M_3(3;-4).$ berilgan. Tomonlari tenglamasi tuzilsin.

№ 33. 1) $\begin{cases} x=z+5 \\ y=4-2z \end{cases}$ va 2) $\begin{cases} \dfrac{x-3}{1}=\dfrac{y-2}{2}=\dfrac{z-3}{1} \end{cases}$ to'g'ri chiziqning xOy va xOz tekisliklardagi izlari topilsin va to'g'ri chiziqlar yasalsin.

Ko'rsatma. to'g'ri chiziqning tenglamalarida 1) z=0 2)y=0 deb faraz qilish kerak.

№ 34. $\left.\begin{matrix} x+2y+3z-13=0 \\ 3x+y+4z-14=0 \end{matrix}\right\}$ to'g'ri chiziq tenglamalarini: 1) proyeksiyalari bo'yicha; 2) kanonik ko'rinishda yozilsin. To'g'ri chiziqning koordinata tekisliklaridagi izlari topilsin hamda to'g'ri chiziq va uning proyeksiyalari yasalsin .

№ 35. A(4;3;0) nuqtadan o'tuvchi va $P\{-1;1;1\}$ vektorga parallel bo'lgan to'g'ri chiziq tenglamari yozilsin. to'g'ri chiziqning yOz tekislikdagi izi topilsin va to'g'ri chiziq yasalsin.

№ 36. x=4 , y=3 to'g'ri chiziq yasalsin va uning yo'naltiruvchi vektori topilsin .

№ 37. 1) $\begin{cases} y=3 \\ z=2, \end{cases}$ 2) $\begin{cases} y=2 \\ z=x+1, \end{cases}$ 3) $\begin{cases} x=4 \\ z=y \end{cases}$ to'g'ri chiziq yasalsin va uning yo'naltiruvchi vektori topilsin.

№ 38. A(-1;2;3) va B(2;6;-2) nuqtalardan o'tuvchi to'g'ri chiziq tenglamalari yozilsin va uning yo'naltiruvchi kosinuslari topilsin .

№ 39. A(2;-1;3) va B(2;3;3) nuqtalardan o'tuvchi to'g'ri chiziq yasalsin va uning tenglamalari yozilsin.

№ 40. A(4;-3;1) nuqtadan chiqib $V\{2;3;1\}$ tezlik bilan harakat qiluvchi M(x;y;z) nuqta traektoriyasining tenglamalari yozilsin.

№ 41. 1) (-2;1;-1) nuqtadan o'tuvchi va $P\{1;-2;3\}$ vektorga parallel bo'lgan;

2) A(3;-1;4) va B(1;1;2) nuqtalardan o'tuvchi to'g'ri chiziqning tenglamalari yozilsin.

V Bob FAZODA TEKISLIK TENGLAMASI.

5.1.Tekislik birinchi tartibli sirt sifatida.

1. Tekislik birinchi tartibli sirt sifatida. 2. Tekislikning chala tenglamalari.
3. Tekislikning kesmalar bo`yicha va normal tenglamalari. 4. Ikki tekislik orasidagi burchak, ikki tekislikning parallelik va erpendikulyarlik shartlari
5. To`g`ri chiziqning yo`naltiruvchi vektorlari. 6. To`g`ri chiziqning kanonik tenglamasi. 7. To`g`ri chiziqning parametrik tenglamalari. 8. Ba'zi tenglamalar va misollar

1. Tekislikda nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa formulasini keltirib chiqaring.
2. To‘g‘ri chiziqlar dastasini turlarini aytib bering va tushuntiring.
3. Ikki to‘g‘ri chiziq orasidagi burchak uchun formula keltirib chiqaring.
4. Ikki to‘g‘ri chiziqning perpendikularlik va parallellik shartlari nimadan iborat?
5. Tekislikda ikki to‘g‘ri chiziq joylashuvini tushuntirib bering.

,B,C}

uning normal vektori bo'lsin. $M_0(x_o, y_o, z_o)$ tekislikning berilgan nuqtasi M(x,y,z) esa uning ixtiyoriy nuqtasi bo'lsin. Demak $\bar{n}\cdot\overline{M_0M}=0\rightarrow$A(x-$x_0$)+B(y-$y_0$)+C(z-$z_0$)=0 (1) berilgan bitta $M_o(x_o, y_o, z_o)$ nuqtadan o'tuvchi tekislik tenglamasi. (1) dan

Ax+By+Cz+(-Ax_o-By_o-Cz_o)=0 ==>Ax+By+Cz+D=0 (2) tekislik tenglamasi. (2)ga tekislikning umumiy tenglamasi deyiladi.

5.2. Tekislikning chala tenglamalari

а) *D=0* да *Ax+By+Cz=0* – koordinata boshidan o'tuvchi tekislik tenglamasi.

б) *C=0* булса *Ax+By+D=0*, demak *n={A,B,O}*. *Shu Ax+By+D=0 Oz* o'qga paratekislik tenglamasi.

в) $B=0$ yoki A=0 bo'lsa $Ax+Cz+D=0$ va $By+Cz+D=0$ lar mos ravishda Oy va Ox o'qlarga parallel tekislik tenglamasi

d) A=B=0 bo'lsa $Cz+D=0$ tenglamaga kelamiz, u Oxy tekisligiga parallel bo'lib, z=-D/C masofadan o'tuvchi tekislik tenglamasidid

e) A=C=0 bo'lsa $By+D=0$ tenglamaga kelamiz, u Oxz tekisligiga parallel bo'lib, y=-D/B masofadan o'tuvchi tekislik tenglamasidir

f) B=C=0 bo'lsa $Az+D=0$ tenglamaga kelamiz, u Ozy tekisligiga parallel bo'lib, z=-D/A masofadan o'tuvchi tekislik tenglamasidid

5.3. Tekislikning kesmalar bo'yicha va normal tenglamalari

Ax+By+Cz=-D tenglamadan $\frac{x}{\frac{-D}{A}}+\frac{y}{\frac{-D}{B}}+\frac{z}{\frac{-D}{C}}=1\Rightarrow\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ kesmalar bo'yicha tekislik tenglamasi. xcosα+ycosβ+zcosγ-p=0 yoki $\frac{Ax+By+Cz+D}{\sqrt{A^2+B^2+C^2}}=0$ tenglamalar tekislikning normal tenglamasidir.

$d=\frac{\left|Ax^*+By^*+Cz^*+D\right|}{\sqrt{A^2+B^2+C^2}}$ formula $M^*(x^*, y^*, z^*)$ nuqtadan Ax+By+Cz+D=0 tekislikgacha bo'lgan masofani topish formulasidir.

5.4. Ikki tekislik orasidagi burchak, ikki tekislikning parallelik va perpendikulyarlik shartlari

Ma'lumki ikki tekislik orasidagi burchak, ularning normal vektorlari orasidagi burchakka teng.

Agar tekisliklar umumiy tenglamalari $\left.\begin{array}{l}A_1x+B_1y+C_1z+D_1=0\\A_2x+B_2y+C_2z+D_2=0\end{array}\right\}$ (1) bilan berilgan bo'lsa, ularning normal vektorlari $\vec{n}_1=\{A_1,B_1,C_1\}$, $\vec{n_2}=\{A_2,B_2,C_2\}$ ko'rinishda bo'ladi. U holda ular orasidagi burchak kosinusi

$$\cos\varphi=\frac{\vec{n_1}\cdot\vec{n_2}}{\left|\vec{n_1}\right|\cdot\left|\vec{n_2}\right|}=\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^{\,2}+B_1^{\,2}+C_1^{\,2}}\cdot\sqrt{A_2^{\,2}+B_2^{\,2}+C_2^{\,2}}}$$ (2) formula bilan aniqlanadi.

Agar ikki tekislik parallel bo'lsa, yoki $\varphi = 0$ bo'ladi, u holda ularning parallelik sharti vector ko'rinishda $\overrightarrow{n_1} = \lambda \cdot \overrightarrow{n_2}$ yoki koordinata shaklida $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} = \lambda$ bo'ladi.

Agar ikki tekislik perpendikulyar bo'lsa, yoki $\varphi = \pi/2$ bo'ladi, u holda ularning perpendikulyarlik sharti vector ko'rinishda $\overrightarrow{n_1} \cdot \overrightarrow{n_2} = 0$ yoki koordinata shaklida $A_1A_2 + B_1B_2 + C_1C_2 = 0$ bo'ladi.

Mavzu bo`yicha takrorlash savollari

1. Fazoda tekislikning berilgan bir nuqtadan o`tuvchi va umumiy tenglamalari?
2. Tekislikning chala tenglamalari?
3. Tekislikning kesmalar bo`yicha va normal tenglamalari?
4. Nuqtadan tekislikgacha masofa hisoblash formulasini yozing

5.5. FAZODA TO`G`RI CHIZIQ.

To`g`ri chiziqning yo`naltiruvchi vektori. To`g`ri chiziqning kanonik tenglamasi. To`g`ri chiziqning parametrik tenglamalari

Biror to`g`ri chiziq berilgan bo`lsin. Shu to`g`ri chiziqda yoki unga parallel to`g`ri chiziqda yotuvchi har bir noldan farqli vektor shu to`g`ri chiziqning yo`naltiruvchi vektori deyiladi.

Faraz qilaylik **a**={l,m,n} vektor to`g`ri chiziqning yo`naltiruvchi vektori bo`lsin. To`g`ri chiziq ustida $M_o(x_0, y_o, z_o)$ (qo`zg`almas) tayin nuqta va M(x,y,z) ixtiyoriy nuqta olaylik. Demak $M_oM=\{x-x_o, y-y_o, z-z_o\}$ va **a**={l,m,n} vektorlar parallel bo`ladi.

Shuning uchun $\frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n}$ (1) shu to`g`ri chiziqning tenglamasi bo`ladi (chunki (1) faqat shu to`g`ri chiziq nuqtalari shundagina to`g`ri bo`ladi. (1) tenglamaga fazodagi to`g`ri chiziqning kanonik tenglamasi deyiladi.

To`g`ri chiziqning parallel bo`lmagan va ustma-ust tushmaydigan teksliklarning kesishish nuqtalaridan iborat deyish mumkin. Faraz qilaylik bunday tekisliklar o`zlarining umumiy tenglamalari bilan berilgan bo`lsin:

$$\left.\begin{aligned} A_1x + B_1y + C_1z + D_1 = 0 \\ A_2x + B_2y + C_2z + D_2 = 0 \end{aligned}\right\} \quad (2)$$

Bu tenglamalar birgalikda to`g`ri chiziqni aniqlaydi. (2) tenglamalar to`g`ri chiziqning umumiy tenglamalari deyiladi. To`g`ri chiziqning umumiy tenglamalaridan uning kanonik tenglamalarini qanday hosil qilish mumkin? Buning uchun (2) ustida bitta nuqta olish kerak. Masalan: $z=z_o$ bo`lsin. x va y larning $x=x_o$ va $y=y_o$ qiymatlarini (2) ni echib topamiz .

Shunday qilib $M_o(x_o, y_o, z_o)$ nuqta hosil bo`ladi (2) ustida. Endi (2) ning yo`naltiruvchi vektor koordinatalarini topish kerak. $\mathbf{n_1}=\{A_1, B_1, C_1\}$ va $n_2=\{A_2, B_2, C_2\}$ vektorlar (2) dagi teksliklarning mos ravishda normal vektorlaridir.

$$\bar{n}_1 \cdot \bar{n}_2 = \left\{\begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix}, \begin{vmatrix} A_1 & C_1 \\ A_2 & C_2 \end{vmatrix}, \begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}\right\}$$

vektor esa (2) to'g'ri chiziqning yo'naltiruvchi vektori bo'ladi. $\mathbf{n_1}\perp\mathbf{n_1}x\mathbf{n_2}$ va $\mathbf{n_2}\perp\mathbf{n_1}x\mathbf{n_2}$), demak (2) to'g'ri chiziqning kanonik tenglamasi

$$\frac{x-x_0}{\begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix}} = \frac{y-y_0}{\begin{vmatrix} A_1 & C_1 \\ A_2 & C_2 \end{vmatrix}} = \frac{z-z_0}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}}$$ bo'ladi

Agar to'g'ri chiziq o'zining kanonik tenglamalari bilan berilgan bo'lsa:

$\frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n} = t$ bulardan $\begin{cases} x = l\cdot t + x_0 \\ y = m\cdot t + y_0 \\ z = n\cdot t + z_0 \end{cases}$ tenglamalarni, ya'ni

to`g'ri chiziqning parametrik tenglamalari deb ataluvchi tenglamalarning hosil qilamiz (t-parametr). To`g`ri chiziqning parametrik tenglamalari yordamida to`g`ri chiziq bilan tekslikning kesishish nuqtasi koordinatalarini topishimiz oson.

Mavzu bo`yicha takrorlash savollari

1. To`g`ri chiziqning yo`naltiruvchi vektorlari?
2. Fazodagi to`g`ri chiziqning kanonik tenglamasini yozing?
3. To`g`ri chiziqning parametrik tenglamalari qanday tenglamalar?
4. To`g`ri chiziqning umumiy tenglamalaridan uning kanonik tenglamalarini qanday hosil qilish mumkin?

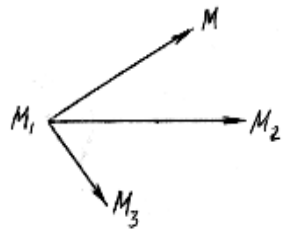
5. To`g`ri chiziqning tekislik bilan kesishish nuqtasini qanday ulash mumkin?

5.6. Ba'zi tenglamalar va misollar

1) Berilgan ikki nuqtadan o`tuvchi to'g'ri chiziq tenglamasi

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1} \qquad (1)$$

2) Berilgan uchta nuqtadan o'tuvchi tekislik tenglamasini yozaylik. Bu nuqtalar $M_1(x_1,y_1,z_1)$, $M_2(x_2,y_2,z_2)$, $M_3(x_3,y_3,z_3)$ bo'lsin, $M(x,y,z)$ tekislikning o'zgaruvchan nuqtasi bo'ladi. $M_1M=\{x-x_1,y-y_1,z-z_1\}$, $M_1M_2=\{x_2-x_1,y_2-y_1,z_2-z_1\}$, $M_1M_3=\{x_3-x_1,y_3-y_1,z_3-z_1\}$ uch vektor komplanar bo'lsa



1 -chizma

$$\begin{vmatrix} x-x_1, y-y_1, z-z_1 \\ x_2-x_1, y_2-y_1, z_2-z_1 \\ x_3-x_1, y_3-y_1, z_3-z_1 \end{vmatrix}=0 \qquad (2)$$

bo'ladi.

Bu berilgan M_1,M_2 va M_3 nuqtalardan o'tuvchi tekislik tenglamasidir. Chunki, agar $M(x,y,z)$ o'zgaruvchan nuqta M_1,M_2,M_3 nuqtalar bilan aniqlanadi, tekislikda yotmasa, u vaqtda (2) tenglama o'rinli bo'lmaydi.

3) Agar $A_1x+B_1y+C_1z+D_1=0$ (I)

$A_2x+B_2y+C_2z+D_2=0$ (II)

tekislik parallel bo'lishi uchun $\mathbf{n_1}=\{A_1,B_1,C_1\}$ va $\mathbf{n_2}=\{A_2,B_2,C_2\}$ lar parallel bo'lishi kerak. Demak $\frac{A_2}{A_1}=\frac{B_2}{B_1}=\frac{C_2}{C_1}$ shart n_1 va n_2 ning parallellik sharti va (I) hamda (II) tekisliklarning parallellik shartidir.

(4) $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ va $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$

to'g'ri shiziqlar parallel bo'lishi uchun $\frac{l_2}{l_1}=\frac{m_2}{m_1}=\frac{n_2}{n_1}$ bo'lishi kerak.

5) Ax+By+Cz+D=0 (*) ва $\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n}$ (**) lar perpendikulyar bo'lishi uchun **n**={A,B,C} va **a**={l,m,n} lar parallel bo'lishi kerak. Shuning uchun $\frac{A}{l}=\frac{B}{m}=\frac{C}{n}$ to'g'ri chiziq va tekislikning perpendikulyarlik shartidir.

6) Agar A·l+B·m+C·n=0 bo'lsa **n** va **a** vektorlar perpendikulyar bo'ladi va demak bu shart (*) va (**) larning parallellik shartidir.

7) (I) va (II) larning perpendikulyarlik sharti esa $A_1A_2+B_1B_2+C_1C_2=0$ dan iborat.

8) To'g'ri chiziqlarning perpendikulyarlik sharti esa $l_1l_2+m_1m_2+n_1n_2=0$ bo'lishi zarur.

Mavzu bo`yicha takrorlash savollari

1. Fazodagi ikki nuqtadan o`tuvchi to`g`ri chiziqning kanonik tenglamalarini yozing?
2. Berilgan uch nuqtadan o`tuvchi tekislik tenglamasini yozing?
3. Ikki tekislikning parallellik va perpendikulyarlik shartlarini yozing?
4. Ikki to`g`ri chiziqning parallellik va perpendikulyarlik shartlarini yozing?
5. Tekislik va to`g`ri chiziqning shartlarini yozing ?

Bobga oid masalalar echish namunalari va mustaqil yechish uchun topshiriqlar

Misol №1. $M_1(2;1;-1;)$ nuqtadan o'tgan va $\vec{n}=\{1;-2;3\}$ normal vektorga ega bo'lgan tekislik tenglamasi tuzilsin.

Yechish: Berilgan $M_0(x_0;y_0z_0)$ nuqtadan o'tuvchi va berilgan $\vec{n}=\{A;B;C\}$ normal vektorga ega tekislik tenglamasi: $A(x-x_0)+B(y-y_0)+c(z-z_0)=0$

$$(x-2)-2(y-1)+3(z+1)=0$$
$$x-2-2y+2+3z+3=0$$
$$x-2y+3z+3=0$$

Misol № 2. $M_1(2;-1;3)$ $M_2(3;1;2)$ nuqtalardan o'tuvchi $\bar{a}=\{3;-1;4\}$. vektorga parallel bo'lgan tekislik tenglamasi tuzilsin.

Yechish: $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l & m & n \end{vmatrix}=0$ $\begin{vmatrix} x-2 & y+1 & z-3 \\ 3-2 & 1+1 & 2-3 \\ 3 & -1 & 4 \end{vmatrix}=0$

8(x-2)-3(y+1)-(z-3)-6(z-3)-4(y+1)-(x-2)=0

7(x-2)-7(y+1)-7(z-3)=0, 7x-7y-7z=0 **J:** x-y-z=0

Misol № 3. $M_1(3;-1;2)$ $M_2(4;-1-1)$ $M_3(2;0;2)$. nuqtalardan o'tuvchi tekislik tenglamasi tuzilsin.

Yechish: $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$ ***J:*** *3x+3y+z-8=0*

№ 4. $M(-1;2;1;)$ $N(2;3;-2;)$ va $P(3;4;2;)$ nuqtalardan o'tuvchi tekislik tenglamasini toping.

№ 5. $M_0(2;0;-3;)$ nuqtadan o'tuvchi va quyidagi shartni qanoatlantiruvchi to'g'ri chiziq tenglamasini tuzing; *a)* $\bar{S}=\{2;3;-4\}$ vektorga parallel; *b)* $M_1(-3;1;4;)$ nuqtadan o'tuvchi;

№ 6. $M_1(2;1;-1;)$ nuqtadan o'tgan va $\vec{n}=\{1;-2;3\}$ normal vektorga ega bo'lgan tekislik tenglamasi tuzilsin.

№ 7. Koordinata boshidan o'tuvchi va $\vec{n}=\{5;0;-3\}$. normal vektorga ega bo'lgan tekislik tenglamasi tuzilsin.

№ 8. $P(2;-1;-1)$ nuqta tekislikka koordinata boshidan tushirilgan perpendikulyar bo'lib, shu tekislik tenglamasi tuzilsin.

№ 9. $M_1(3;-1;2)$ $M_2(4;-2;-1)$. nuqtalar berilgan. M_1 nuqtadan o'tuvchi, $\overline{M_1M_2}$ vektorga perpendikulyar bo'lgan tekislik tenglamasi tuzilsin.

№ 10. $M_1(3;4;-5)$ nuqtadan o'tuvchi $\bar{a}_1=\{3;1;-1\}$ $\bar{a}_2=\{1;-2;1\}$. vektorlarga parallel bo'lgan tekislik tenglamasi tuzilsin.

$$\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

№ 11. $M_1(2;-1;3)$ $M_2(3;1;2)$ nuqtalardan o'tuvchi $\bar{a}=\{3;-1;4\}$. vektorga parallel bo'lgan tekislik tenglamasi tuzilsin.

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l & m & n \end{vmatrix}=0$$

№ 12. $M_1(3;-1;2)$ $M_2(4;-1-1)$ $M_3(2;0;2)$. nuqtalardan o'tuvchi tekislik tenglamasi tuzilsin. $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$

№ 13.

1)2x-y-2z+5=0 2)x+5y-z=0 3)3x-2y-6=0 4)5y-3z=0 5)x+2=0 6)y-3=0 Tekisliklar yasalsin va unga normal vektorning koordinata o'qlari bilan tashkil etgan burchaklari topilsin.

№ 14. Qaysi juft tekisliklar parallel tekisliklar ekanligini aniqlang.

1)2x-3y+5z-7=0 , 2x-3y+5z+3=0; 2)4x+2y-4z+5=0 2x+y+2z-1=0; 3)x-3z+2=0 2x-6z-7=0.

№ 15. Qaysi juft tekisliklar perpendikulyar tekisliklar ekanligini aniqlang.

1)3x-y-2z-5=0, x+9y-3z+2=0; 2)2x+3y-z-3=0 , x-y-z+5=0 3)2x-5y+z=0 , x+2z-3=0

№ 16. m va n лар qaysi hollarda parallel tekisliklarni beradi: 1) 2x+my+3z-5=0, nx-6y-6z+2=0; 2)3x-y+mz-9=0, 2x+ny+2z-3=0; 3)mx+3y-2z-1=0 2x-5y-nz=0.

№ 17. m qaysi hollarda perpendikulyar tekisliklarni beradi.

1) 3x-5y+mz-3=0, x+3y+2z+5=0; 2)5x+y-3z=0, 2x+my-3z+1=0 3)7x-2y-z=0, mx+y-3z-1=0

№ 18. Quyidagi tekisliklar o'zaro qanday joylashgan?

1) x-y$\sqrt{2}$ +z-1=0, x+y$\sqrt{2}$ -z+3=0; 2)3y-z=0, 2y+z=0

3) 6x+3y-2z=0, x+2y+6z-12=0; 4) x+2y+2z-3=0, 16x+12y-15z-1=0.

№ 19. Koordinata boshidan o'tuvchi va 5x-3y+2z-3=0 tekislikka paralell bo'lgan tekislik tenglamasi tuzilsin.

№ 20. M_1(3;-2;-7) nuqtadan o'tuvchi 2x-3z+5=0 tekislikka parallel bo'lgan tekislik tenglamasi tuzilsin.

№ 21. Koordinata boshidan o'tuvchi va 2x-y+3z-1=0, x+2y+z=0 tekisliklarga perpendikulyar bo'lgan tekislik tenglamasi tuzilsin.

№ 22. M_1(2;-1;1) nuqtadan o'tuvchi 2x-z+1=0 , y=0 tekisliklarga perpendikulyar bo'lgan tekislik tenglamasi tuzilsin.

$$\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix} = 0$$

№ 23. 1)x-2y+2z-8=0 ва x+z-6=0; 2)x+2z-6=0 va x+2y-4=0 tekisliklar orasidagi burchak topilsin.

№ 24. (2;2;-2) nuqtadan o'tuvchi va x-2y-3z=0 tekislikka parallel tekislik topilsin

№ 24-1. (-1;-1;2) nuqtadan o'tuvchi va x-2y+z-4=0 hamda x+2y-2z=0 tekisliklarga perpendikulyar tekislikning tenglamasi yozilsin.

№ 25. (0;0;a) nuqtadan o'tuvchi va x-y-z=0 hamda 2y=x tekisliklarga perpendikulyar tekislikning tenglamasi yozilsin.

№ 26. M_1(-1;-2;0) va M_2(1;1;2) nuqtalardan o'tuvchi hamda x+2y+2z-4=0 tekislikka perpendikulyar tekislikning tenglamasi yozilsin.

№ 27. M_1(1;-1;2) ,M_2(2;1;2) va M_3(1;1;4) nuqtalardan o'tuvchi tekislikning tenglamasi yozlsin. .

№ 28. Oz o'qdan 2x+y-$\sqrt{5}$z =0 tekislik bilan 60° burchak tashkil etuvchi tekislik o'tkazilsin.

№ 29. (5;1;-1) nuqtadan x-2y+2z+4=0 tekislikkacha bo'lgan masofa topilsin.

№ 30. (4;3;0) nuqtadan M_1(1;3;0) , M_2(4;-1;2) va M_3(3;0;1) nuqtalardan o'tuvchi tekislikkacha bo;lgan masofa topilsin.

№ 31. 4x+3y-5z-8=0 va 4x+3y-5z+12=0 parallel tekislik orasidagi masofa topilsin.

№ 32. 1)x-2y+2z-5=0 tekislikka parallel va undan 2 birlik uzoqlikda bo'lgan tekislik lar tenglamalari yozilsin.

№ 33. 1)2x-y+3z-6=0 va x+2y-z+3=0 tekislikning kesishgan chizig'idan va (1;2;4) nuqtadan o'tuvchi tekislik tenglamasi yozilsin.

2) x=y va z=0 tekisliklarning kesishgan to'g'ri chizig'idan o'tuvchi o'zaro perpendikulyar ikkita tekislikdan bittasi (0;4;2) nuqtadan ham o'tadi. To'gri chiziq va izlangan tekisliklar topilsin.

№ 34. 2x-y+3z-9=0; x+2y+2z-3=0; 3x+y-4z+6=0 tekisliklarning kesishgan nuqtasi topilsin.

№ 35. (2;-1;1) nuqtadan o'tuvchi va $3x+2y-z+4=0$ va $x+y+z-3=0$ tekisliklarga perpendikulyar tekislikning tenglamasi yozilsin. Tekislik yasalsin.

№ 36. (0;-5;0) va (0;0;2) nuqtalardan o'tuvchi hamda $x+5y+2z-10=0$ tekislikka perpendikulyar tekislik ning tenglamasi yozilsin. Tekislik yasalsin.

№ 37. Ox o'qdan o'tuvchi va $y=x$ tekislik bilan 60^0 burchak tashkil etuvchi tekislikning tenglamasi yozilsin.

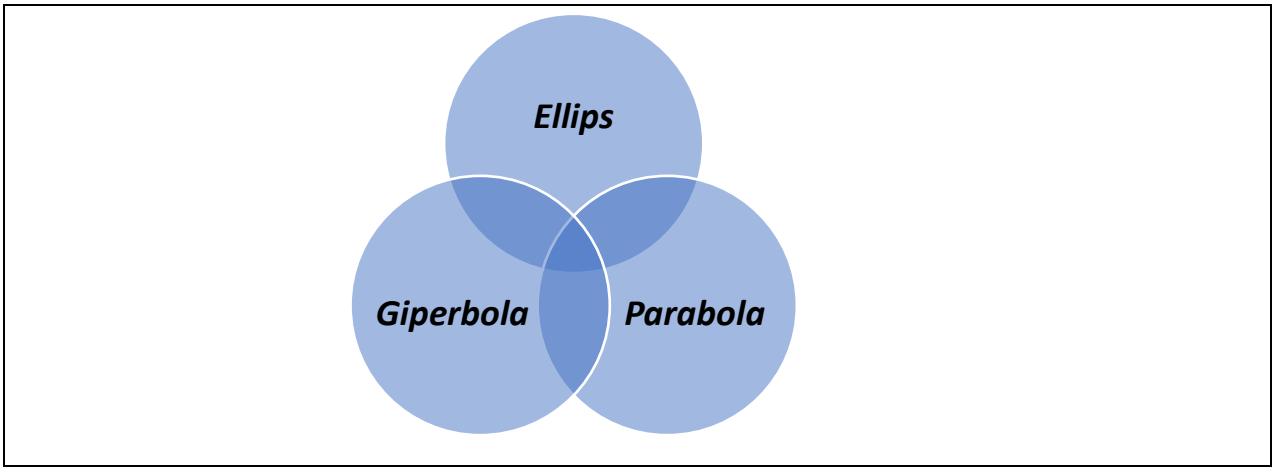
№ 38. $2x+2y+z-8=0$ tekislikka parallel va undan $d=4$ masofada bo'lgan tekisliklarning tenglamalari yozilsin.

№ 39. $4x-y+3z-6=0$ va $x+5y-z+10=0$ tekisliklarning kesishgan chizig'idan o'tuvchi va $2x-y+5z-5=0$ tekislikka perpendikulyar tekislikning tenglamasi yozilsin.

5.7. IKKINCHI TARTIBLI EGRI CHIZIQLAR.

1. Aylana va uning tenglamasi. 2. Ellips, uning kanonik tenglamasini keltirib chiqarish. 3. Ellips ekstsentriteti. 4. Ellips fokal radiuslari uchun ratsional ifodalar. 5. Ellipsni nuqtalar bo`yicha yasash. 6. Ellipsning parametrik tenglamalari

Venn diagrammasi



(3-ilova)

T-sxema

Ikkinchi tartibli egri chiziqlar	Bir hil nomlanuvchi tushunchalar	Ular orasidagi farqlar
Ellips		
Giperbola		
Parabola		

5.7.1. Oldindan berilgan chiziq tenglamasini keltirib chiqarish

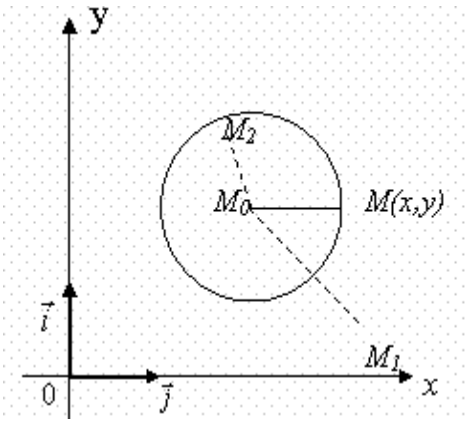
Markazi C(α;β) nuqtada va radiusi **r** bo'lgan aylana tenglamasi

CM= r =√(x-α)²+(y-β)² yoki (x-α)²+(y-β)²=r². Endi A(2;0) nuqtaga harakati davomida B(8;0) nuqtaga qaraganda ikki baravar yaqin turadigan M(x;y) nuqta traektoriyasining tenglamasini chiqaring. 2·AM=BM bo'lgani uchun, $AM = \sqrt{(x-2)^2 + y^2}$, $BM = \sqrt{(x-8)^2 + y^2}$. Demak

$$2 \cdot \sqrt{(x-2)^2 + y^2} = \sqrt{(x-8)^2 + y^2}$$

$$4x^2 - 16x + 16 + 4y^2 = x^2 - 16x + 64 + y^2 \Rightarrow 3x^2 + 3y^2 = 48 \Rightarrow x^2 + y^2 = 16 = 4^2$$

- aylana.



1-chizma

1-misol: $x^2 + y^2 + 2x - 6y + 1 = 0$ tenglamani kanonik ko'rinishga keltiring hamda aylananing $M_0(x_0,y_0)$ markazi va r radiusini toping.

Yechish: $(x^2 + 2x + 1) + (y^2 - 6y + 9) - 9 = 0 \Rightarrow (x+1)^2 + (y-3)^2 = 9$.

Oxirgi tenglikda $x = X - 1,\ y = Y + 3$ almashtirishlarni bajaramiz.

$$X^2 + Y^2 = 9 \Rightarrow M_0(-1,\ 3),\ r = 3.$$

Javob: $(x+1)^2 + (y-3)^2 = 9,\ M_0(-1,\ 3),\ r = 3.$

2-misol: Markazi $M_0(1,2)$ nuqtada bo'lib, $6x + 8y - 15 = 0$ to'g'ri chiziqqa uringan aylana tenglamasini tuzing.

Yechish: M_0 nuqtadan berilgan to'g'ri chiziqqacha bo'lgan masofani topamiz:

$r = d(M_0, l) = \dfrac{|6 \cdot 1 + 8 \cdot 2 - 15|}{10} = \dfrac{7}{10}$. Demak $(x-1)^2 + (y-2)^2 = \dfrac{49}{100}$.

Bobga oid masalalar echish namunalari va mustaqil yechish uchun topshiriqlar

№ 1. Quyidagi hollarda aylana tenglamasini tuzing:

1) Aylana markazi koordinatalar boshidan o'tadi va uning radiusi R=3; J: $x^2+y^2=9$

2) Aylana markazi C(2;-3) nuqtadan o'tadi va radiusi R=7; J: $(x-2)^2+(y+3)^2=49$

3) Aylana koordinatalar boshidan o'tadi va uning markazi C(6;-8) nuqtaga mos keladi; J: $(x-6)^2+(y+8)^2=100$

4) Aylana A(2;6) nuqtadan o'tadi va uning markazi C(-1;2) nuqtaga mos keladi;

J: $AC=R=\sqrt{(-1-2)^2+(2-6)^2}=\sqrt{25}=5$ $(x+1)^2+(y-2)^2=25$

5)A(3;2) va B(-1;2) nuqtalar aylananing biror bir diametrining oxiri bo'ladi;

J: $O(\alpha;\beta)$ $AB=2R=\sqrt{(-1-3)^2+(6-2)^2}=4\sqrt{2}$ $R=2\sqrt{2}$

6) Aylana markazi koordinatalar boshi bilan ustma-ust tushuvchi va 3x-4y+20=0 to'g'ri chiziq aylanaga urinma bo'lgan;

J: $x_0=0$ $y_0=0$ $A=3$ $B=-4$ $C=20$ *O(0;0)* nuqtadan L to'g'ri chiziqqacha bo'lgan masofa $d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}=\frac{|20|}{5}=4$ $x^2+y^2=16$

7) Aylana markazi C(1;-1) nuqta bilan ustma-ust tushadi va 5x-12y+9=0 to'g'ri chiziq aylanaga urinma bo'lgan;

J: $x_0=1$ $y_0=-1$ $A=5$ $B=-12$ $C=9$ *C(1;-1)* nuqtadan L to'g'ri chiziqqacha bo'lgan masofa $d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}=\frac{|26|}{13}=2$ $(x-1)^2+(y+1)^2=4$

№ 2. Markazi C(-4;3) , radiusi R=5 bo'lgan aylana tenglamasi yozilsin va u yasalsin. A(-1;1), B(3;2), O(0;0) nuqtalar bu nuqtada yotadimi?

№ 3. A(-4;6) nuqta berilgan. Diametri OA kesmadan iborat aylana tenglamasi yozilsin.

№ 4. 1)$x^2+y^2-4x+6y-3=0$; 2)$x^2+y^2-8x=0$; 3)$x^2+y^2+4y=0$ aylanalar yasalsin .

№ 5. $x^2+y^2+5x=0$ aylana va $x+y=0$ to'g'ri chiziq yasalsin va ularning kesishgan nuqtalari topilsin.

№ 6. A(1;2) nuqtadan o'tuvchi va koordinata o'qlariga urinuvchi aylana tenglamasi yozilsin.

№ 7. $x^2+y^2+4x-6y=0$ aylananing Oy o'q bilan kesishgan nuqtalariga o'tkazilgan radiuslari orasidagi burchak topilsin.

№ 8. A(-1;3) , B(0;2) va C(1;-1) nuqtalardan o'tuvchi aylana tenglamasi yozilsin.

Ko'rsatma. Izlanayotgan aylananing tenglamasini $x^2+y^2+mx+ny+p=0$ ko'rinishda yozib, undagi x va y lar o'rniga berilgan har bir nuqtaning koordinatalarini qo'ygandan so'ng m, n ва p larni topish kerak.

№ 9. A (4;4) nuqtadan va $x^2+y^2+4x-4y=0$ aylana bilan y = -x to'g'ri chiziqning kesishgan nuqtalaridan o'tuvchi aylana tenglamasi yozilsin.

№ 10. $y=-\sqrt{-x^2-4x}$ egri chiziqning joylashish sohasi aniqlanib, shakli chizilsin.

№ 11. $x^2+y^2-8x-4y+16=0$ aylanaga koordinatalar boshidan o'tkazilgan urinmalarning tenglamalari yozilsin.

№ 12. A (a;0) nuqta berilgan. M nuqta shunday harakat qiladiki, ΔOMA da OMA burchak doimo to'g'ri burchak bo'lib qoladi. M nuqta traektoriyasining tenglamasi yozilsin.

№ 13. A (-6;0) va B (2;0) nuqtalar berilgan. Shunday nuqtalarning geometrik topilsinki, ulardan. OA va OB kesmalar teng burchak ostida ko'rinsin.

Ko'rsatma. O'zgaruvchi nuqtani M deb olsak, OM kesma AMB burchakning bissektrisasi bo'ladi. Izlngan tenglamani chiqarish uchun uchburchak ichki burchagining bissektrisasi qarshi tomonni bo'lishi haqidagi teoremadan foydalanish kerak.

№ 14. A(-3;0) va B(3;6) nuqtalar berilgan. Diametri AB kesmadan iborat aylana tenglamasi yozilsin.

№ 15. 1) $x^2+y^2-6x+4y-23=0$; 2) $x^2+y^2+5x-7y+2,5=0$; 3) $x^2+y^2+7y=0$ aylanalarning markazlari va radiuslari topilsin. Aylanalar yasalsin.

№ 16. Aylana A(0;-4) nuqtadan o'tadi va koordinatalar boshida Ox o'qqa urinadi. Aylana tenglamasi yozilsin va unig koordinata burchaklarining bissektrisalari bilan kesishgan nuqtalari topilsin.

№ 17. A(1;-2), B(0;-1) va C(-3;0) nuqtalardan o'tuvchi aylanaga koordinatalar boshidan o'tkazilgan urinmalar tenglamalari yozilsin.

№ 18. $x^2+y^2-4x+6y-5=0$ aylananing Ox o'q bilan kesishgan nuqtalariga o'tkazilgan radiuslari orasidagi burchak topilsin.

№ 19. A(3;0) nuqta $x^2+y^2-4x+2y+1=0$ aylana ichida yotishi ko'rsatilsin va A nuqtada teng ikkiga bo'linuvchi vatar tenglamasi yozilsin.

Ko'rsatma. Izlanuvchi vatar CA ga perpendikulyardir, bunda C- aylana markazi.

№ 20. Quyidagi hollarda aylana tenglamasini tuzing:

1) Aylana markazi koordinatalar boshidan o'tadi va uning radiusi R=3;

2) Aylana markazi C(2;-3) nuqtadan o'tadi va radiusi R=7;

3) Aylana koordinatalar boshidan o'tadi va uning markazi C(6;-8) nuqtaga mos keladi;

4) Aylana A(2;6) nuqtadan o'tadi va uning markazi C(-1;2) nuqtaga mos keladi;

5) A(3;2) va B(-1;2) nuqtalar aylananing biror bir diametrining oxiri bo'ladi;

6) Aylana markazi koordinatalar boshi bilan ustma-ust tushuvchi va 3x-4y+20=0 to'g'ri chiziq aylanaga urinma bo'lgan;

7) Aylana markazi C(1;-1) nuqta bilan ustma-ust tushadi va 5x-12y+9=0 to'g'ri chiziq aylanaga urinma bo'lgan;

8) Aylana quyidagi nuqtalardan o'tadi A(3;1) ва B(-1;3) ,uning markazi esa 3x-y-2=0 to'g'ri chiziqda yotadi;

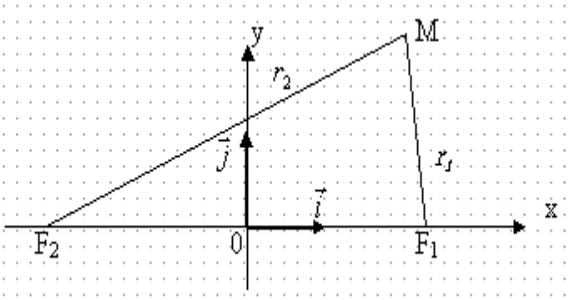
9) Aylana quyidagi uchta nuqtadan o'tadi A(1;1) , B(1;-1) ,C(2;0);

№ 21. C(3;-1) nuqta aylana markazi hisoblanadi. Aylanani kesib o'tuvchi 2x-5y+18=0 to'g'ri chiziqning vatari uzunligi 6 ga teng. Bu aylananing tenglamasini tuzing.

№ 22. $M_1(3;1)$ nuqtadan o'tuvchi va radiusi R=$\sqrt{5}$ gat eng bo'lgan, x-2y-1=0 to'g'ri chiziqqa urinuvchi aylana tenglamasi yozilsin.

5.7.2. ELLIPS

Ta'rif: Ellips deb ikkita fokuslar deb ataluvchi qo`zg`almas F_1 va F_2 nuqtalargacha bo`lgan masofalarining yig`indisi o`zgarmas 2a songa teng bo'lgan M(x,y) nuqtalarning geometrik o`rinlariga aytiladi.



Faraz qilaylik M-ellipsning ixtiyoriy nuqtasi, F_1 va F_2 - fokuslari $F_1M+F_2M=2a$, $F_1F_2=2c$, $2a>2c$ ==> $a>c$; F_1 (-c;0) ; F_2 (c;0)

$r_1=\sqrt{(x+c)^2+y^2}$, $r_2=\sqrt{(x-c)^2+y^2}$, $r_1+r_2=2a=const$,

$\sqrt{(x+c)^2+y^2}+\sqrt{(x-c)^2+y^2}=2a$ ellips tenglamasi.

Bu tenglamani soddalashtirib ellipsning kanonok tenglamasini topamiz.

$x^2+2cx+c^2+y^2-4a\cdot\sqrt{(x+c)^2+y^2}+4a^2=x^2-2xc+c^2+y^2$

$4a^2+4cx=4a\cdot\sqrt{(x+c)^2+y^2}+a^2+cx=a\cdot\sqrt{(x+c)^2+y^2}$

$a^4+2a^2cx+c^2x^2=a^2x^2+2a^2c+a^2c^2+a^2y^2$ $(a^2-c^2)\cdot x^2+a^2y^2=a^2(a^2-c^2)$;

$\frac{x^2}{a^2}+\frac{y^2}{a^2-c^2}=1$, $a^2-c^2=b^2$; $a^2-b^2=c^2$. (1) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ - ellipsning kanonik tenglamasi.

(1) tenglamadan $y=\pm\frac{b}{a}\cdot\sqrt{a^2-x^2}, y=\frac{b}{a}\cdot\sqrt{a^2-x^2}$ egri chiziqni qaraymiz. Agar x=0 bo'lsa y=b bo'ladi. Agar x bo'lsa y va x=a bo'lganda y=0 bo'ladi. (*) dan ko'rinib turibdiki ellipsning koordinata tizimiga nisbatan va koordinata boshiga nisbatan simmetrik egri chiziqdir. A,B,C,D ellips uchlari deyiladi. O-markaz 2a va 2b – ellips o'qlari, a va b katta va kichik yarim o'qlar. Agar b=a bo'lsa $x^2+y^2=a^2$ -aylanadir.

Ellips ekssentrisiteti

$\varepsilon=\frac{2c}{2a}$ yoki $\varepsilon=\frac{c}{a}$ - ellips ekssentrisiteti deyiladi. $\varepsilon^2=\frac{c^2}{a^2}=\frac{a^2-b^2}{a^2}=1-\left(\frac{a}{b}\right)^2$ va $\varepsilon=\sqrt{1-\left(\frac{a}{b}\right)^2}$, agar b<<a bo'lsa $\varepsilon\approx1$ va ellips absissa bo'ylab cho'zilgan bo'ladi.

Agarda b≈a bo'lsa $\varepsilon\approx0$ bo'ladi va ellips shakli aylana shakliga yaqin bo'ladi. Demak ε-ellips shaklini xarakterlaydi. Ellips uchun

$0<\varepsilon<1$ bo'lar ekan.

Ellips fokal radiuslari uchun rasional ifodalar

Oldinroqdagi (A) formulaga $a\cdot\sqrt{(x+c)^2+y^2} = a^2+cx$ dan $r_1=\sqrt{(x+c)^2+y^2}$

Bo'lgani uchun $r_1=a+\frac{c}{a}\cdot x=a-\varepsilon\cdot x$ dan esa $r_2=a-\varepsilon\cdot x$ formula hosil bo'ladi.

$r_1=a+\varepsilon\cdot x$ ва $r_2=a-\varepsilon\cdot x$ formulalar ellips radiuslari uchun ratsional ifodali formuladir

Ellipsni nuqtalar bo'yicha yasash.
Ellipsning parametric tenglamalari

$$\left.\begin{array}{ll} \frac{x}{a}=const \Rightarrow x=a\cdot\cos t & x=a\cdot\cos t \\ \frac{y}{b}=\sin t \Rightarrow y=b\cdot\sin t & y=b\cdot\sin t \end{array}\right\}$$ - ellipsning parametrik tenglamalari, bu

erda t-parametr.

Misol № 1. $9x^2+25y^2=225$ 1) yarim o'qlarini; 2) uning fokuslarini 3) eksstentrisiteti va 4) direktrisa tenglamalari topilsin.

J: 1) $\frac{9x^2}{225}+\frac{25y^2}{225}=1 \quad \frac{x^2}{25}+\frac{y^2}{9}=1 \quad a=5 \;\; b=3$

2) $c^2=a^2-в^2=25-9=16 \quad c=4$

3) ekstsentrisiteti: $\varepsilon=\frac{c}{a}=\frac{4}{5}$

4) direktrissalari: $x=\pm\frac{a}{\varepsilon}=\pm\frac{5}{\frac{4}{5}}=\pm\frac{25}{4}$

Misol № 2. Agar ellipsning:1) katta yarim o'qi $a=10$, eksstentrisiteti $\varepsilon=0{,}8$ bo'lsa, uning kanonik tenglamasi yozilsin.

J: $2a=10 \;\; a=5 \;\; \varepsilon=\frac{c}{a}\Rightarrow c=a\cdot\varepsilon=5\cdot 0{,}8=4 \quad в^2=a^2-c^2$ dan foydalanib,

$в=\sqrt{5^2-4^2}\Rightarrow в=3 \quad \frac{x^2}{a^2}+\frac{y^2}{b^2}=1\Rightarrow \;\; \frac{x^2}{25}+\frac{y^2}{9}=1$

Mavzu bo`yicha takrorlash savollari

1. Ellipsning ta'rifini keltiring?
2. Ellipsning kanonik tenglamasi kanday tenglama?
3. Ellipsning ekstsentrisiteti nima?
4. Ellipsning fokal radiuslari uchun ratsional ifodalar yozing?
5. Ellips nuqtalar bo`yicha qanday yasaladi?
6. Ellipsning parametrik tenglamalarini yozing?

Bobga oid masalalar echish namunalari va mustaqil yechish uchun topshiriqlar

№ 1. $x^2+4y^2=16$ ellips yasalsin, uning fokuslari va eksstentrisiteti topilsin.

№ 2. Agar ellipsning:1) fokuslari orasidagi masofa 8 ga teng bo'lib, kichik yarim o'qi b=3; katta yarim o'qi a=6, eksstentrisiteti $\varepsilon = 0{,}5$ bo'lsa, uning kanonik tenglamasi yozilsin.

№ 3. Ellips katta yarim o'qi a=5 va c parametri: 1) 4,8; 2) 4; 3) 3; 4) 1,4; 5) 0 ga teng bo'lsa, uning kichik yarim o'qi $в$ va ekssentrisiteti ε topilsin.

№ 4. Koordinata o'qlariga nisbatan simmetrik bo'lgan ellips $M(2;\sqrt{3})$ va B(0;2) nuqtalardan o'tadi. Uning tenglamasi yozilsin va M nuqtadan fokuslargacha bo'lgan masofalar topilsin.

№ 5. Fokuslari Ox o'qda yotuvchi ellips koordinata o'qlariga nisbatan simmetrik bo'lib, $M(-4;\sqrt{21})$nuqtadan o'tadi va $\varepsilon = \frac{3}{4}$ eksstentrisitetiga ega. Ellips tengalamasi yozilsin va M nuqtaning fokal radiuslari topilsin.

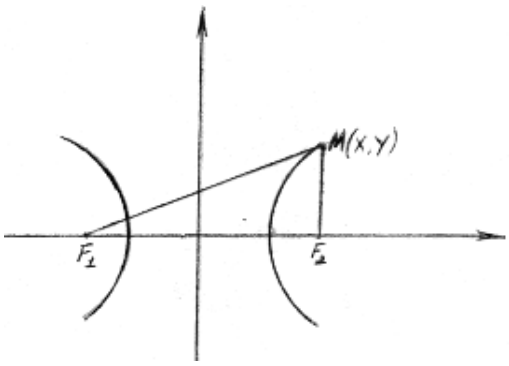
№ 6. $x^2+2y^2=18$ ellipsning o'qlari orasidagi burchakni teng ikkiga bo'luvchi vatar uzunligi topilsin.

5.7.2. GIPERBOLA

Giperbola va uning kanonik tenglamasi shakli. Giperbola ekstsentrisiteti. Ellips va giperbola direktrisalari. Parabola va uning kanonik tenglamasi.

Ta'rif: Giperbola deb ikkita fokuslar deb ataluvchi qo`zg`almas F_1 va F_2 nuqtalargacha bo`lgan masofalarning ayrimasi o`zgarmas 2a son bo`lgan M(x,y) nuqtalarning geometrik o`rniga aytiladi. $F_1(-c;0)$; $F_2(c;0)$- fokuslar

$$r_1 = \sqrt{(x+c)^2+y^2}, r_2 = \sqrt{(x-c)^2+y^2} \qquad r_1 - r_2 = 2a = const.$$



1-chizma

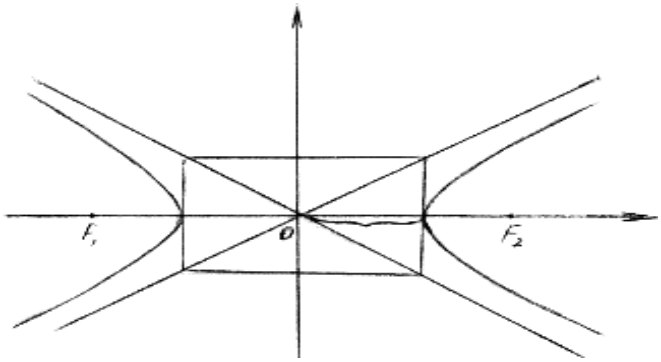
$\sqrt{(x+c)^2+y^2}-\sqrt{(x-c)^2+y^2}=\pm 2a$ - Giperbola tenglamasi; 2a<2c ; a<c 2a=F_1M-F_2M<F_1F_2=2c Giperbola tenglamasini soddalashtirib $\frac{x^2}{a^2}-\frac{y^2}{c^2-a^2}=1$ va $c^2-a^2=b^2$ desak (*) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ tenglamani hosil qilamiz. (*) tenglama giperbolaning kanonik tenglamasi deyiladi. (*) dan $y=\pm\frac{a}{b}\sqrt{x^2-a^2}, y=\frac{a}{b}\sqrt{x^2-a^2}$ -giperbola o'ng tarmog'i. $y=\frac{b}{a}\cdot x$ - to'g'ri chiziq, giperbola asimptotasi.

$$Y-y=\frac{b}{a}\cdot x-\frac{b}{a}\sqrt{x^2-a^2}=\frac{b}{a}\cdot(x-\sqrt{x^2-a^2})=\frac{b}{a}\cdot\frac{x^2-(x^2-a^2)}{x+\sqrt{x^2-a^2}}=$$

agar x$\to\infty$ bo'lsa.

$$\frac{b}{a}\cdot\frac{a^2}{x+\sqrt{x^2-a^2}}=\frac{ab}{x+\sqrt{x^2-a^2}}\to 0,$$

Giperbolani chizish uchun asosiy to`rtburchakni chizish kerak, diagonallarini chizish kerak va giperbola chizig`ini chizish kerak.



2- chizma

Giperbola ekssentrisiteti

$$E=\frac{c}{a}, E^2=\frac{c^2}{a^2}=\frac{a^2+b^2}{a^2}1+\left(\frac{b}{a}\right)^2; E=\sqrt{1+\left(\frac{b}{a}\right)^2}$$

Agar b<<a bo'lsa a ε ≈1 bo'ladi va giperbola tarmoqlari absissa o'qiga yaqinlashib, keladi. Agarda b>>a bo'lsa giperbola shakli to'g'rilanadi (to'g'ri chiziqqa yaqinlashadi). Xullas ekssentrisitet giperbola shaklini harakterlaydi. Giperbola uchun ε >1.

Ellips va giperbola direktrisalari

Ox o'qga perpendikulyar bo'lib koordinata boshidan o'ng va chap tomonda $x=\frac{a}{\varepsilon}$ va $x=-\frac{a}{\varepsilon}$ tenglamalar bilan aniqlanib o'tuvchi to'g'ri chiziqlar ellips va giperbola direktrisalari deyiladi. Ellips uchun 0< ε <1 bo'lgani uchun $\frac{a}{\varepsilon}>a$; giperbola uchun esa ε >1 bo'lganidan $\frac{a}{\varepsilon}<a$ bo'ladi

Teorema. Agar r ellipsning biror nuqtasidan uning biror fokusigacha bo'lgan masofa bo'lsa, hamda d o'sha nuqtadan shu fokusga mos direktrisagacha bo'lgan masofa bo'lsa, u vaqtda $\frac{r}{d}=\varepsilon$ bo'ladi. Shu mazmundagi teorema giperbola uchun ham o'rinli bo'ladi.

1-misol: Giperbola $M_1(4,6)$ nuqtadan o'tib, haqiqiy o'qi 4 ga teng bo'lsa, kanonik tenglamasi yozilsin.

Yechish: $2a=4\Rightarrow a=2.\quad \frac{x^2}{4}-\frac{y^2}{b^2}=1\Rightarrow \frac{16}{4}-\frac{36}{b^2}=1\Rightarrow b^2=12.$

$$Жавоб: \frac{x^2}{4}-\frac{y^2}{12}=1.$$

2-misol: $x^2-4y^2+2x+16y-7=0$ giperbolaning markazi va yarim o'qlarini toping.

Yechish: $\left(x^2+2x+1\right)-4\left(y^2-4y+4\right)+8=0$

$$(x+1)^2-4(y-2)^2+8=0\Rightarrow \frac{(y-2)^2}{2}-\frac{(x+1)^2}{8}=1.$$

Giperbolaning haqiqiy o'qi OU to'g'ri chiziqqa parallel bo'lib, markazi (-1,2) nuqtada, yarim o'qlari $b=\sqrt{2},\ a=\sqrt{8}$ ga teng.

3-misol: $c=6,\ e=\frac{6}{5}$ bo'lgan giperbolaning kanonik tenglamasi yozilsin

Yechish: $e=\frac{c}{a}=\frac{6}{5}\Rightarrow \frac{6}{a}=\frac{6}{5}\Rightarrow a=5.$

$c^2-a^2=b^2\Rightarrow b^2=36-25=9.$ $\qquad Жавоб: \frac{x^2}{25}-\frac{y^2}{9}=1.$

Bobga oid masalalar echish namunalari va mustaqil yechish uchun topshiriqlar

№ 1. $x^2-4y^2=16$ giperbola va uning assimptotalari yasalsin. Giperbolaning fokuslari , ekssentrisiteti va assimptotalari orasidagi burchak topilsin.

№ 2. $x^2-4y^2=16$ giperbolada ordinatasi 1ga teng M nuqta olingan.Undan fokuslargacha bo'lgan masofalar topilsin.

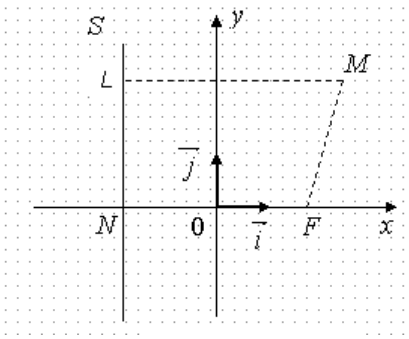
№ 3. *a*) Fokuslari orasidagi masofa 2c=10, uchlari orasidagi masofa 2a=8; б) haqiqiy yarim o'qi $a=2\sqrt{5}$, ekssentrisiteti $\varepsilon=\sqrt{1,2}$ bo'lgan giperbolaning kanonik tenglamasi yozilsin.

№ 4. Giperbola koordinata o'qlariga nisbatan simmetrik bo'lib, $M(6;-2\sqrt{2})$ nuqtadan o'tadi va $\varepsilon=2$ mavhum yarim o'qqa ega . Uning tenglamasi yozilsin hamda M nuqtadan fokuslargacha bo'lgan masofalar topilsin.

5.7.3. PARABOLA

Ta'rif: Har bir nuqtasidan fokus deb ataluvchi berilgan nuqtagacha va direktrisa deb ataluvchi berilgan to'g'ri chiziqqacha bo'lgan masofa o'zaro teng bo'lgan tekislikdagi barcha nuqtalar o'rniga parabola deb ataladi.

Parabola fokusini F va direktrisasini S orqali belgilaylik. F fokus S direktrisaga tegishli emas. $d(F,S)=p$ bo'lsin. F nuqtadan S to'g'ri chiziqqacha perpendikulyar o'tkazamiz va uni absissa o'qi Ox uchun olamiz. $d(F,N)=p$ bo'lib, $N\in S$. FN kesmaning o'rtasi O nuqtadan OX ga perpendikulyar ordinata o'qi OU o'tkazamiz. Oxy-Dekart koordinatalar sistemasini tashkil etadi. O'rnatilgan Dekart koordinatalar sistemasida F va S nuqtalar $F\left(\frac{P}{2},0\right)$, $N\left(-\frac{P}{2},0\right)$ koordinatalarga ega.



Direktrisaning tenglamasi $x=-\frac{p}{2}.$, fokus $F\left(\frac{P}{2},0\right)$ koordinatalarga ega. Parabola

ta’rifiga ko’ra

$$d(F,M)=d(L,M) \qquad (1)$$

(1) tenglikni koordinatalar bo’yicha kanonik holga keltiraylik. $B=\left\{0,\vec{i},\vec{j}\right\}$ reperda M nuqta M(x,u) koordinatalarga ega bo’lsin. ,

$$d(F,M)=\sqrt{\left(x-\frac{P}{2}\right)^2+y^2},\quad d(L,M)=\sqrt{\left(x+\frac{P}{2}\right)^2}=x+\frac{P}{2}.$$

$$\sqrt{\left(x-\frac{P}{2}\right)^2+y^2}=\left|x+\frac{P}{2}\right| \qquad (2)$$

(2) tenglamaning har ikki tomonini kvadratga ko’taramiz:

$$x^2-px+\frac{p^2}{4}=x^2+px+\frac{x^2}{4}\Rightarrow y^2=2px \qquad (3)$$

(3) tenglama (1) tenglikdan kelib chiqadi. Endi (3) dan foydalanib, (1) tenglikni isbotlaymiz. Ixtiyoriy $M_1(x_1,u_1)$ nuqta (3) tenglamani qanoatlantirsin: $y_1^2=2px_1$. $F\left(\frac{p}{2},0\right)$ nuqta va to’g’ri chiziqni olaylik.

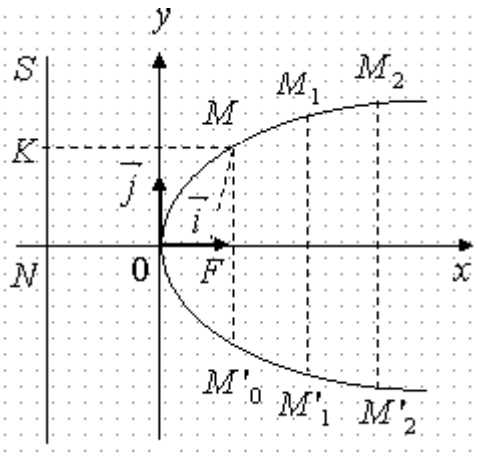
$$d(F,M)=\sqrt{\left(x_1-\frac{P}{2}\right)^2+y_1^2}=\sqrt{x_1^2-Px_1+\frac{P^2}{4}+2Px_1}=\sqrt{x_1^2+Px_1+\frac{P^2}{4}}=$$

$$=\sqrt{\left(x_1+\frac{P}{2}\right)^2}=\left|x_1+\frac{P}{2}\right|=d(M_1,S)=d(L,M_1).$$

Ko’ramizki, M_1 nuqta parabolaga tegishli. (3) ni parabolaning kanonik tenglamasi deyiladi. Parabola shaklini aniqlaylik.

1) Parabola ikkinchi tartibli chiziq.
2) $y^2\geq 0$ va $p>0\Rightarrow x\geq 0.$ Bundan parabolaning barcha nuqtalari x≥0 yarim tekislikka tegishligi kelib chiqadi.
3) $x=0,\ y=0\Rightarrow$ parabola koordinatalar boshidan o’tadi. O(0,0) nuqtani parabolaning uchi deyiladi.
4) x ning har bir $x>0$ qiymatiga u ning qarama-qarshi ishorali, ammo absolyut miqdorlari teng bo’lgan ikki qiymati mos keladi. Xulosa shuki, parabola OX o’qqa nisbatan simmetrik chiziq.

5) $y^2 = 2Px \Rightarrow y = \pm\sqrt{2Px} \Rightarrow x \to \infty \Rightarrow |y| \to \infty.$ Yuqoidagi mulohazalar bo'yicha parabolaning shakli quyidagicha bo'lishi mumkin. (9- chizma). Agar parabola fokusi $F(-\frac{P}{2},0)$ bo'lsa, u holda uning tenglamasi $y^2 = -2px$ (4) ko'rinishida bo'ladi. Agar fokus Oy o'qining $E\left(0,\frac{p}{2}\right)$ nuqtasi bo'lsa, u holda parabolaning tenglamasi $x^2 = 2Py$ ko'rinishida bo'ladi.



Parabolani quyidagi usul bilan yasash mumkin. $y^2 = 2px$ kanonik tenglamasi bilan berilgan parabolaning fokusi va direktrisasini yasaymiz. Buning uchun OX o'qida O nuqtadan chapda va o'ngda $d(N,O) = D(O,F) = \frac{p}{2}$ bo'lgan N va F nuqtalarni belgilaymiz. N nuqtadan OX ga perpendikulyar s to'g'ri chiziq o'tkazamiz. F fokusdan boshlab OX o'qqa perpendikulyar va har biri oldingisidan $P/2$ masofada turuvchi to'g'ri chiziqlarni o'tkazamiz. O'tkazilgan to'g'ri chiziqlarning har biridan direktrisagacha bo'lgan masofani radius qilib F markazli aylana chizamiz. Bu aylana mos to'g'ri chiziqni OX o'qqa simmetrik bo'lgan ikki nuqtada kesib o'tadi. Bu nuqtalar izlangan parabolaga tegishli bo'ladi. OX o'qqa perpendikulyar istalgancha to'g'ri chiziqlarni o'tkazish mumkin. Shularning har birida ta'rifni qanoatlantiruvchi parabola nuqtalarini aniqlaymiz. Aniqlangan nuqtalarni tutashtirib parabola grafigini hosil qilamiz.

1-misol. $x^2 = -12y$ parabolada fokal radiusi $r = 9$ bo'lgan nuqta topilsin.

Yechish: $2q = -12 \Rightarrow q = -6.\ F(0,3).\ r = \sqrt{x^2 + (y+3)^2} = 9 \Rightarrow x^2 + (y+3)^2 = 81.$

$$(y+3)^2 - 12y - 81 = 0 \Rightarrow y^2 - 6y - 72 - 0 \Rightarrow y_1 = -6,\ y_2 = 12.$$

Bu sonlarni $x^2 = -12y$ tenglamaga qo'ysak, $x_1 = -12 \cdot (-6) = 72,\ x_{1,2} = \pm 6\sqrt{2}.$ $y_2 = 12$ tenglamani qanoatlantirmaydi, chunki $x^2 = -12 \cdot 12 = -144.$ $x_{3,4} = \pm 12i$ (mavhum son)

Javob: $M_1\left(6\sqrt{2},-6\right),\ M_2\left(-6\sqrt{2},6\right).$

2-misol: $y=4x^2-6x-\frac{3}{4}$ parabola tenglamasini kanonik ko'rinishga keltirilsin va parabola uchining koordinatalari aniqlansin.

Yechish: $y=4\left(x^2-\frac{3}{2}x+\frac{9}{16}\right)-\frac{9}{4}-\frac{3}{4}\Rightarrow y=4\left(x-\frac{3}{4}\right)^2-3\Rightarrow y+3=4\left(x-\frac{3}{4}\right)^2.$

$y+3=Y,\ x-\frac{3}{4}=X$ belgilashlarni kiritsak, $Y=4X^2.$ Parabolaning uchun $O'\left(\frac{3}{4},-3\right)$ nuqtada. $\left(O'Y\right)-$simmetriya o'qi. Javob: $Y=4X^2,\ O'\left(\frac{3}{4},-3\right).$

3-misol: $x^2-6x+9-4\left(y+1\right)=0$ tenglama koordinatlar boshini ko'chirish orqali kanonik ko'rinishga keltirilsin. Almashtirish formulasi yozilsin.

Yechish: $x^2-6x+9-4\left(y+1\right)=0\Rightarrow\left(x-3\right)^2=4\left(y+1\right).$

$x-3=X,\ y+1=Y$ ko'rinishidagi belgilash kiritsak, $X^2=4Y, O'\left(3,-1\right).$

Javob: Simmetriya o'qi $O'(3,-1)//\left(OY\right)$ bo'lgan parabola. Uchi $O'(3,-1)$ nuqtada, parametri $p=\frac{1}{8}.$ Almashtirish formulalari $x=X+3,\ y=Y-1.$

Mavzu bo`yicha takrorlash savollari

1. Giperbolaning ta'rifini keltiring?
2. Giperbolaning kanonik tenglamasi qanday?
3. Giperbolaning assimptotalari qanday chiziqlar?
4. Giperbolani chizish uchun uning asosiy to`rtburchagini chizish kerak. Bu qanday turtburchak?
5. Giperbola ekstsentristeti nima?
6. Ellips va giperbola direktrissalari nima?
7. Parabola ta'rifini keltiring?
8. Parabolaning kanonik tenglamasini yozing?
9. Ikkinchi tartibli egri chizqilarning *texnikada qanday qo`llanishlari bor?*

«Blits-so’rov» savollari savol-javob vaqtida to’ldiriladi

№	Savollar	Javoblar
1.	Ikkinchi tartibli egri chiziqlarning umumiy ko’rinishdagi tenglamasini yozing?	
2.	Umumiy tenglama koeffitsientlari qanday munosabatlarni qanoatlantirganda, ikkinchi tartibli egri chiziq aynan aylanani aniqlaydi?	
3.	Tekislikda qanday nuqtalar to’plamiga ellips deyiladi?	
4.	Ellipsning kanonik shakldagi tenglamasini yozing va uni izohlang?	
5.	Ellipsning uchlari, yarim o’qlari, simmetriya markazi va simmetriya o’qlarini ko’rsating?	
6.	Ellipsning ekssentrisiteti deb qanday kattalikka aytiladi va u nimani xarakterlaydi?	
7.	Qanday ellipsga aylana deyiladi?	
8.	Tekislikda qanday nuqtalar to’plamiga giperbola deyiladi?	
9.	Giperbolaning kanonik shakldagi tenglamasini yozing va uni sharhlang?	
10.	Giperbolaning haqiqiy va mavhum uchlarini, yarim o’qlarini ko’rsating, simmetriya markazi, simmetriya o’qlari va asosiy to’g’ri to’rtburchagi mavjudmi?	

VI bob
VEKTORLAR HAQIDA ASOSIY TUSHUNCHALAR

1- §. Asosiy tushunchalar va vektorning ta'rifi

Ma'lumki ko'pgina fizik kattaliklar vektor miqdorlar bo'lib, ular nafaqat o'zlarining son qiymatlari balki yo'lalishlari bilan ham xarakterlanadilar, shuning uchun ularni son qiymatlari va yo'nalishlarini aniqlash - ko'rsatish zarur. Vektor miqdorning geometric modeli yo'naltirilgan kesma bo'lib, uning uzunligi vektorning con qiymatini, yo'lalishi esa uning yo'lalishini ko'rsatadi.

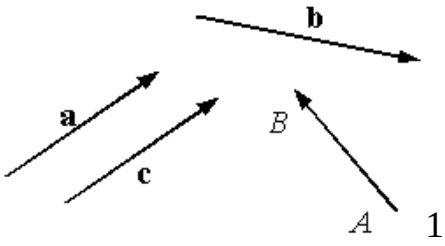
Harorat, massa, zichlik kabilar skalyar miqdorlar, kuch, nuqtaning siljishi, tezlik, tezlanish kabilar esa vektor miqdorlardir. Har bir skalyar miqdor biror son bilan xarakterlanadi.

Vektor miqdor esa son berilishi bilan to`liq tavsiflanmaydi, chunki vektor miqdorlar o`lchamlilikdan tashqari yana yo`nalishga ham ega. Matematikada erkin vektorlar qaraladi, ya'ni ularni tekislik yoki fazoning bir nuqtasidan ikkinchi nuqtasiga kattaligi - uzunligi hamda yo'nalishini o'zgartirmasdan ko'chirish mumkin.

Ta'rif O'zinig son qiymati va yo'nalishi bilan xarakterlanadigan miqdorlarga vector miqdorlar deyiladi yoki yo'nalishga ega kesma vector deb aytiladi.

Demak, vector bu kesma – va uning *vektorning uchi_(oxiri*) deb ataluvchi bitta oxirgi nuqtasi, ikkinchu uchi esa uning *boshlahg'ch nuqtasi* deyiladi.

Matematik adabiyotlarda vector **a**, $\bar{a}$, $\vec{a}$, $\overline{AB}$, $\overrightarrow{AB}$ ko'rinishlarda biri shaklida belgilanadi. Ikkita oxirgi bilgilashlarda A vektorni boshlang'ch nuqtasini, B eca oxirgi nuqtasini bildiradi



1-cizma Vektorlarni giometrik tasviri.

Ta’rif. Vektorlar ***teng*** deyiladi, agar ularga mos kesmalar parallel hamda bir xil kattalik va yo’nalishga ega bo’lsalar.

Agar 1-chizmadagi vektorlar bitta tekislikda yotadi deb faraz qilsak, u holda **a** va **c** vektorlar teng $\bar{a} = \bar{c}$, demak bunda **a** va **c** bitta vektorni turlicha belgilanishdir.

Vektorlar algebrasida erkin vektorlar bilah ish ko’riladi, yuqoridagi vektorlarning tengligi ta’rifidan vektorni tekislik yoki fazoning bir nuqtasidan ikkincni nuqtasiga o’ziga parallel ko’chirish mumkin va bunda vector o’zgarmaydi.

Ta’rif Vektorlar ***kollinear*** vektorlar deyiladi, agar ular bir to`g`ri chiziqda yoki parallel to`g`ri chiziqlarga parallel yoki ularda yotsalar.

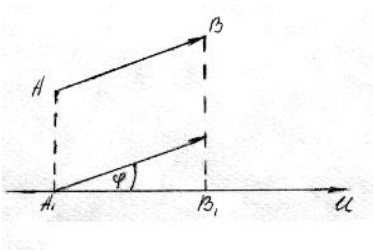
Ta’rif Agar uchta vektorlar bir tekislikda yoki parallel tekisliklarda yotsalar ular ***komplanar*** vektorlar deyiladi.

Ta’rif Vektorning uzunligi yoki moduli deb unga mos yo’naltirilgan kesma uzunligiga aytiladi.

a vektorning uzunligi yoki moduli $|\mathbf{a}|$ kabi belgilanadi. **a** vector ***birlik*** vector deyiladi agar, $|\mathbf{a}|=1$ tenglik o’rinli bo’lsa.

Vektorlar to’plamiga bitta ob’ekt qo’shish zarur, biz uni ***nol vector*** deb ataymiz. Bu vektorni boshlang’ch va oxirgi nuqqtalari ustma-ust tushuvchi kesma deb qarash mumkin. Bunday vektorning uzunligi nolga teng, u yo’nalishga ega emas. Barcha nol vektorlar bir biriga teng. Nol vector ixtiyoriy to’g’ri chiziqda yotishini inobatga olsak, ta’rifga asosah u har qanday vektorga ***kolleniar*** va ***perpendikulyar***dir.

Vektorlar nazariyasida qabul qilingan belgilgshlarga ko’ra nok vektorni **0** kabi belgilash kerak, lekin uni oddiy qilib 0 deb belgilanadi.



Vektorlar yo`naltirilgan kesma shaklida belgilanadi. $\overline{AB}$ (A boshi, B esa oxiri) va **a** shaklda belgilanadi. Agar vektorning boshi va oxiri bir nuqtada bo`lsa u vektor nol vektor deyiladi.

2-chizma

Agar vetorlar kollinear bo`lsa, bir xil uzunlikda bo`lsa va bir xil yo`nalgan bo`lsa ular teng vektorlar deyiladi. Biz erkli vektorlarnigina qaraymiz, ya'ni vetor

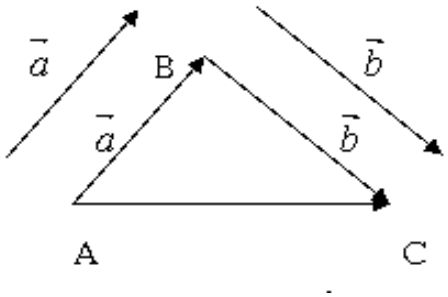
uzunligi va yo`nalishini o`zgartirmagan holda uning boshini xoxlagan nuqtada o`rnatilgan deb faraz qilish mumkin.

Vektor uzunligi uning moduli deyiladi va ¦a¦ shaklda belgilanadi. Tushunarliki **a**=**b** dan ¦**a**¦=¦**b**¦ ekani kelib chiqadi. Biror **u** o'q berilgan bo'lsin $\overline{AB}$ =**a** bo'lsin. $\text{пр}_u \overline{AB}$ =A'B'=|$\overline{AB}$|·cosφ=|**a**|·cosφ bo'ladi.

2 - §. Vektorlar ustida chiziqli amallar

Vektorlarni qo`shish va vektorlarni songa ko`paytirish amallari vektorlar ustida chiziqli amallar deyiladi.

1) Qo`shish. **a** va **b** vektorlar berilgan bo`lsa **a+b** vektor deb, agar **a** ning oxiriga **b** qo`yilgan bo`lsa, **a** ning boshidan **b** ning oxiriga keluvchi vektorga aytiladi. (shakliga qarang, uchburchak qoidasi.)

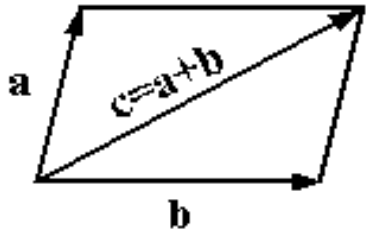


3-chizma (uchburchak qoidasi)

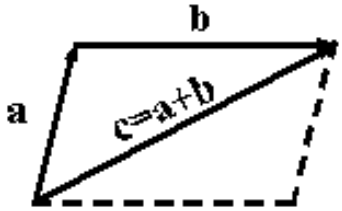
2) Songa vektorni ko`paytirish. **a** vektor va λ son bo'lsin. λ**a** (yoki **a**λ) ko'paytma vektordir. Bu vektor **a** ga kollinear, uzunligi | λ |**a**|| ga teng, yunalishi λ >0 bo'lganda **a** bilan bir xil, λ <0 bo'lganda esa **a** yo`nalishiga teskari.

Agar λ =0 bo'lsa **a**λ =0 - nol vektor deyiladi (yo`nalishi aniq emas) Aslida **a** ni **a** ga ko`paytirishni **a** ni a marta "uzunlashtirish"dir.

Tarif Ikkita **a** va **b** vektorlaring yig'ndisi deb shunday uchinchi **c** vektorga aytiladiki, ularning uchlarini bir nuqtaga keltirganda **a** va **b** vektorlar parallellogramning tomonlari **c** vektor esa uning diognali bo'ladi.

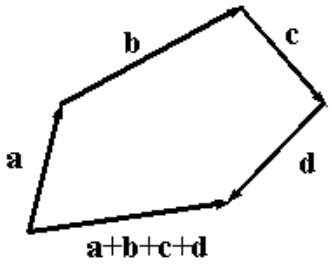


4-chizma (parallellogram qoidasi)



5-chizma (uchburchak qoidasi)

4- va 5- chizmalardan ko'rish mumkinki vektorlarni uchburchak yoki parallelogram qoidalari asosida qo'shganda bir xil natija hosil bo'lar ekan.



Chekli sondagi vetorlar yig'ndisi ko'pburcha qoidasiga asosan topiladi.

6-chizma (ko'pburchak qoidasi)

Ta'rif. **b** vector **a** vektorga *qarama - qarshi* vector deyiladi, agar **a** va **b** vektorlar kolleniar, yo'nalishlari qarama - qarshi **va** $|\mathbf{a}|=|\mathbf{b}|$ tenglik o'rinli bo'lsa. **a** vektorga *qarama - qarshi* vector $-\mathbf{a}$ kabi belgilanadi, yoki $\overrightarrow{AB}=-\overrightarrow{BA}$

Ta'rif. Ikkita **a** va **b** vektorlaring *ayirmasi* deb $\mathbf{a}+(-\mathbf{b})$ yig'ndiga aytiladi va ayrma $\mathbf{a}-\mathbf{b}$ kabi belgilanadi, yoki $\mathbf{a}-\mathbf{b}=\mathbf{a}+(-\mathbf{b})$ tenglik o'rinli bo'ladi.

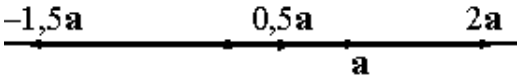
Ta'rif. **a** *vektorni* α *haqiqiy songa* ko'paytmasi deb, quyidagi shartlar bilan aniqlanuvchi **b** vektorga aytiladi,

1) $|\mathbf{b}|=|\alpha|\cdot|\mathbf{a}|$ va, agar $|\mathbf{b}|\neq 0$ bo'lsa, quyodagi ikkita shart ham o'rinli bo'ladi:

2) **b** vector **a** vektorga kolleniar;

3) agar $\alpha>0$ bo'lsa **b** va **a** vektorlar bir xil yo'nalishda, agar, $\alpha<0$ bo'lsa qarama-qarshi yo'nalishda bo'ladilar

a *vektorni* α *haqiqiy songa* ko'paytmasi $\alpha\mathbf{a}$ kabi belgilanadi.



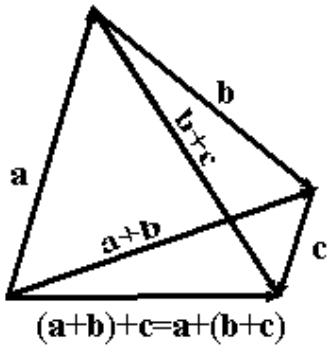
7-chizma

Vektorlarni qo’shish va vektorni songa ko’paytirishning ba’zi xossalarini qarab chiqaylik. Ularning ba’zilari vector tushunchasini umumashtirishda zarur bo’ladi, ularni terema k’rinishida keltiraylik.

Teorema. *Har qanday* $\mathbf{a},\mathbf{b},\mathbf{c}$ *vektorlar va haqiqiy* α,β *sonlar uchun, quyidagi xossalar o’rinli:*

1) $\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}$ (*qo’shishga nisbatan kommutativlik xossasi*);
2) $(\mathbf{a}+\mathbf{b})+\mathbf{c}=\mathbf{a}+(\mathbf{b}+\mathbf{c})$ (*qo’shishga nisbatan assosativlik xossasi*);
3) $\mathbf{a}+0=\mathbf{a}$;
4) $\mathbf{a}+(-\mathbf{a})=0$;
5) $\alpha(\beta\mathbf{a})=(\alpha\beta)\mathbf{a}$ (*skalyar miqdorlarga nisbatan assosativlik xossasi*);
6) $\alpha(\mathbf{a}+\mathbf{b})=\alpha\mathbf{a}+\alpha\mathbf{b}$ (*songa k’paytirishga nisbatan distirbutivlik xossasi*);
7) $(\alpha+\beta)\mathbf{a}=\alpha\mathbf{a}+\beta\mathbf{a}$ (*vektorga k’paytirishga nisbatan distirbutivlik xossasi*);
8) $1\cdot\mathbf{a}=\mathbf{a}$.

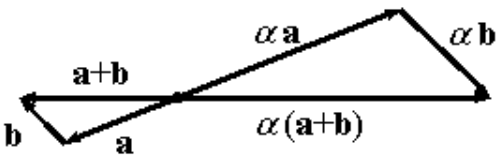
Teoremani ba’zi xossalarini isbotini keltiramiz, qolganlari ham ularga o’xshab oson isbotlanadi.



8-chizma

Ikkinchi qo’shishga nisbatan assosativlik xossaini isboti ushbu chizmadan kelib chiqadi.

Songa k’paytirishga nisbatan distirbutivlik xossasini ushbu chizmadan kelib chiqadi.



9-chizma

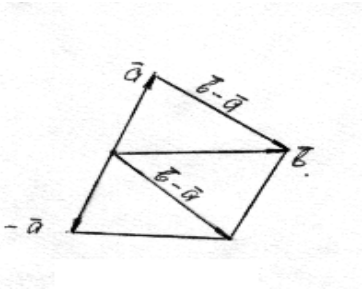
Endi vektorlarni qo’shish va songa k’paytirishning sodda xossalarni isbotsiz keltiramiz.

9-xossa $\alpha\mathbf{a}=0$ tenglik, $\alpha=0$, yoki $\mathbf{a}=0$ da o'rinli;

10-xossa **a** vektorga qarama-qarshi vector, $(-1)\cdot\mathbf{a}$ ga teng, ya'ni $-\mathbf{a}=(-1)\cdot\mathbf{a}$;

11-xossa Har qanday ikkita **a** va **b** vektorlar uchun shunday **x** vector mavjudki, uning uchun $\mathbf{a}+\mathbf{x}=\mathbf{b}$ tenglik o'rinli bo'ladi.

1.2.1 Chiziqli amallarning xossalari



10-chizma

1) $\bar{a}+\bar{b}=\bar{b}+\bar{a}$ **A**gar **a** va **b** lar umumiy boshga keltirilgan bo'lsa **a+b** shu vektorlarga qurilgan parallelogramm diagonalidir (umumiy boshdan chiqqan). Shu kabi uchta, to'rtta va hokazo vektorlar yig`indisini ham topib bo`ladi. **a,b,c** vektorlar xar qanday bo`lganda ham $(\bar{a}+\bar{b})+\bar{c}=\bar{a}+(\bar{b}+\bar{c})$ bo'ladi.

3 - §. Vektorlar sistemasining chiziqli bog'liqligi va bog'liqmasligi

Vektorlar sistemasining chiziqli bog'liqligi va bog'liqmasligi tushunchalarini qarab chiqayli.

Ta'rif 1. $\vec{a}_1, \vec{a}_2, \vec{a}_3, \ldots, \vec{a}_n$ vektorlar sistemasi chiziqli bog'liq deyiladi, agar ularning chiziqli kombinatsiyasi nolga teng bo'lsa, ya'ni

$$a_1\overrightarrow{a_1} + a_2\overrightarrow{a_2} + \cdots + a_n\overrightarrow{a_n}=0$$

va chiziqli kombinatsiya koefitsienlari bir vaqtda nolga teng emas. Bu holda qanaydir vektorni boshqalarining chiziqli kombinatsiya orqali ifodalash mumkin bo'ladi. Masalan, agar $a_1 \neq 0$ bo'lsa, u holda $\overrightarrow{a_1} = \alpha\overrightarrow{a_2} + \beta\overrightarrow{a_3} + \cdots + \delta\overrightarrow{a_n}$ tenglik o'rinli bo'ladi.

Masalan: ikkita chiziqli bog'liq $\vec{a}$ va $\vec{b}$ vektorlar, $\vec{\boldsymbol{a}} = \boldsymbol{\delta} * \vec{\boldsymbol{b}}$, tenglik bilan berilgan bo'lsa, ularni chiziqli kombinatsiyasi $1*\vec{a} - \delta * \vec{b} = 0$ ko'rinishda bo'ladi. Bunday vektorlar kolleniardir.

Uchta chiziqli bog'liq vektorlar $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ uchun

$$\boldsymbol{a_1\vec{a} + a_2\vec{b} + a_3\vec{c}=0}$$

tenglik o'rinli bo'ladi. Bundan uchta chiziqli bog'liq vektorlar $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ birortasini qolgan ikkitasining chiziqli kombinatsiyasi orqali ifodalash mumkin, ya'ni $\vec{a} = \alpha \vec{b} + \beta \vec{c},$

Bu holda uchta vector ham bir tekislikda yotadi, hamda uchunchi vector $\alpha * \vec{a}$ va $\beta * \vec{b}$ vektorlarga qurilgan parallelogramnig dionaligi bo'ladi.

Ta'rif 2

$\vec{a}_1, \vec{a}_2, \vec{a}_3, \ldots, \vec{a}_n$ vektorlar sistemasi chiziqli bog'liqmas deyiladi, agar ularning chiziqli kombinatsiyasi nolga teng bo'lmasa, yoki ulardan hech birini qolganlari chiziqli kombinatiyasi orqali ifodalash mumkin bo'lmasa.

Chiziqli bog'liq bo'lmagan vektorlarning chiziqli kombinatsiyasi nolga teng bo'lishi mumkin, faqat agar ularnig chiziqli kombinatsiyasida barcha koeffisientlarning qiymatlari bir vaqtda nolga teng bo'lsa.

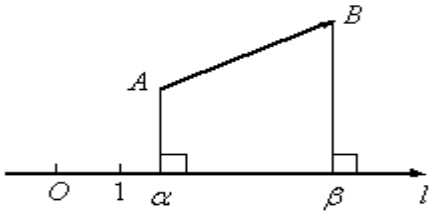
3.1. *Vektorning o'qqa va koordinata o`qlariga proeksiyalari*

Faraz qilaylik, tekislik yoki fazoda biror l chiziq berilgan bo'lib unda tayin O nuqta, o'quning musbat yo'nalishi hamda uzunlik birligi mashstab berilgan bo'lsin. U holda o'qning har bir nuqtasiga bitta son mos keladi.

Ta'rif. A nuqtaning l chiziqdagi poeksiyasi deb undan o'qqf tushurilgan AB perpendikulyar asosiga mos keluvchi songa aytiladi.

Ta'rif. $\overrightarrow{AB}$ venorning l chiziqdagi poeksiyasi deb, uning oxirgi va boshlang'ch nuqtalari poeksiyalari orasidagi farqga aytiladi.

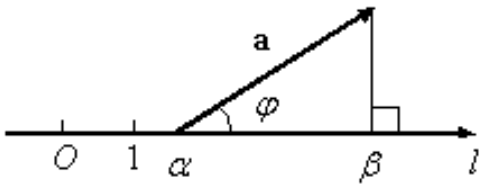
Venorning l chiziqdagi poeksiyasi $pr_l \overline{AB}$ deb belgilanadi va $pr_l \overline{AB} = \beta - \alpha$ ga teng. Agar $\overrightarrow{AB} = \overrightarrow{CD}$ bo'lsa, $pr_l \overline{AB} = pr_l \overline{CD}$ bo'ladi.



11 - chizma

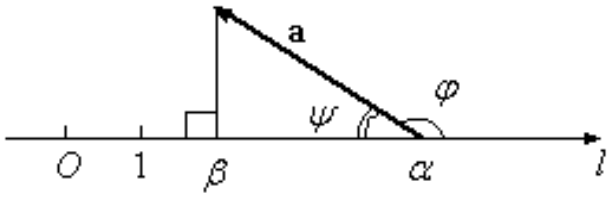
Teorema *φ - chiziq va a vector orasidagi burchak bo'lsn. U holda* $pr_l\overline{AB}=|a|\cos\varphi$ *bo'ladi.*

Isbot. Faraz qilaylik φ -o'tkir burchak bo'lsin. U holda quyidagi chizmadan $\beta-\alpha=|\mathbf{a}|\cos\varphi$ ekanligi kelib chiqadi.



12 - chizma

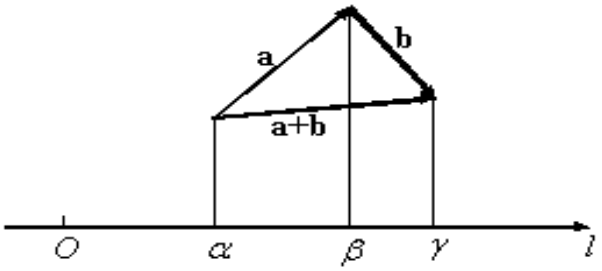
Agar φ o'tmas bolsa pastdagi chizmadan, $\alpha-\beta=-|\mathbf{a}|\cos\psi$ ni aniqlaymiz, bundan $\beta-\alpha=|\mathbf{a}|\cos\varphi$.



13 - chizma

Teorema. Vektorlar yig'ndisini o'qqa proeksiyasi ular proeksiyalari yig'ndisiga teng.

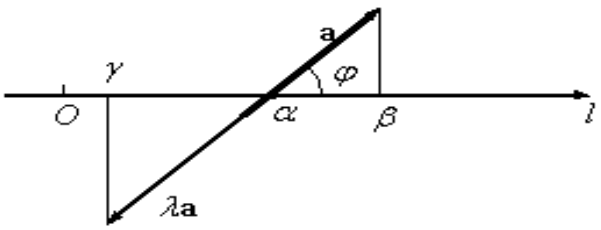
Agar qo'shiluvchi vektorlar proeksiyalari bir xil ishorali bo'lsa teorema isboti keying chizmadan ko'rinib turibdi.



14 – chizma

Teorema. Biror bir songa ko'paytirilgan vektorning o'qga proyeksiyasi, vector poeksiyasini shu songa ko'paymasiga teng, ya'ni $pr_l\lambda\overline{AB}=\lambda pr_l\overline{AB}$.

Teorema isboti uchburchaklar o'xshashlik alomatidan bevosita kelib chiqadi, pastdagi rasmga qaralsin.



15 - chizma

Ta'rif. **b** *vektorning* **a**, $\mathbf{a} \neq 0$ vektorga proeksiyasi deb, **b** vektorni **a** vektorga parallel va u bilan bir xil yo'nalishga ega bo'lgan o'qga proeksiyasiga aytiladi va $pr_{\bar{a}}\bar{b}$ deb belgilanadi, $pr_{\bar{a}}\bar{b} = |\bar{b}|\cos\varphi$ tenglik o'rinli, bunda φ **a** va **b** vektorlar orasidagi burchak.

Fazoda to'g'ri burchakli Dekart koordinata sistemasi berilgan bo'lsin. **a** vektorning koordinata o'qlaridagi proektsiyalari X, Y, Z bo`lsin. Bu holda **a**={X,Y,Z} deb yoziladi

3.2. Proeksiyalar haqida teoremalar

Teorema 1. $\Pi p_u (\mathbf{a}_1 + \ldots + \mathbf{a}_n) = \Pi p_u \mathbf{a}_1 + \ldots + \Pi p\, \mathbf{a}_n$

Teorema 2. $\Pi p\, (\alpha \cdot \mathbf{a}) = \alpha \cdot \Pi p_u \mathbf{a}$

Demak **a**={X_1, Y_1, Z_1} va **b**={X_2,Y_2,Z_2 } bo'lsa **a**±**b**={X_1+X_2;Y_1+Y_2;Z_1+Z_2} va $\alpha\cdot\mathbf{a}$ ={$\alpha\cdot X_1$, $\alpha\cdot Y_1$,$\alpha\cdot Z_1$} bo'ladi

Endi ikki vektorning kollinearlik shartini topamiz. **a** = {X_1, Y_1, Z_1} va **b**={X_2,Y_2,Z_2 } bo'lsin. Vektorlar kollinear bo'ladi, agar **b**= $\lambda\cdot\mathbf{a}$ bo'lsa (a≠0), yoki $X_2=\lambda\cdot X_1$, $Y_2=\lambda\cdot Y_1$, $Z_2=\lambda\cdot Z_1$ bo'ladi . Bundan esaikki vektorning kollinearlik sharti $\frac{X_2}{X_1} = \frac{Y_2}{Y_1} = \frac{Z_2}{Z_1}$ -ekanligi kelib chiqadi.

Masala $\bar{a} = \{5;2;5\}$ vektorni $\bar{b} = \{2;-1;2\}$ vektordagi proyeksiyasini toping.

Yechish. $\bar{a}\cdot\bar{b} = |\bar{b}|\cdot pr_{\bar{b}}\bar{a}$; $\bar{a}\cdot\bar{b} = 5\cdot 2 + 2\cdot(-1) + 5\cdot 2 = 18$; $|\bar{b}| = \sqrt{4+1+4} = 3$;

$pr_{\bar{b}}\bar{a} = \frac{\bar{a}\cdot\bar{b}}{|\bar{b}|} = \frac{18}{3} = 6$; Javob: 6.

Masala $\bar{a} = \{1;-3;4\}$; $\bar{b} = \{3;-4;2\}$ va $\bar{c} = \{-1;1;4\}$ vektorlar berilgan. $pr_{\bar{b}+\bar{c}}\bar{a}$ ni hisoblang.

Yechish.

$\bar{b}+\bar{c} = \{2;-3;6\}$; $|\bar{b}+\bar{c}| = \sqrt{4+9+36} = 7$; $\bar{a}\cdot(\bar{b}+\bar{c}) = |\bar{b}+\bar{c}|\cdot pr_{\bar{b}+\bar{c}}\bar{a}$;

$1\cdot 2+(-3)(-3)+4\cdot 6=7\cdot pr_{\bar{b}+\bar{c}}\bar{a}$; Javob: 5.

Teorema A(x_1, y_1, z_1) va B(x_2,y_2 ,z_2) nuqtalar har qanday bo'lganda ham AB vektor koordinatalari

$$X=x_2-x_1,\ Y=y_2-y_1,\ Z=z_2-z_1 \qquad (1)$$

formula bilan aniqlanadi.

Vektorlar erkli bolgani uchun uning boshi koordinata boshida deb faraz qilish mumkin. Agar **a**={X,Y,Z} bo'lsa, $|\mathbf{a}|^2=x^2+y^2+z^2$ (Pifagor teoremasiga asosan). Demak $|a|=\sqrt{x^2+y^2+z^2}$ bo'ladi.

Natija Vektorning koordindta o'qlariga proeksiyalari uning koordindtalariga teng.

1-Masala. $|\bar{a}|=13;\ |\bar{b}|=1;$ va $|\bar{a}+\bar{b}|=24;$ berilgan. $|\bar{a}-\bar{b}|$ ni hisoblang.

Yechish.

1- ***usul.***

$|\bar{a}+\bar{b}|$ - parallelogramni katta dioganali; $|\bar{a}-\bar{b}|$ - kichik dioganaloni beradi.

Parallelogrammni tomonlari va dioganallari o'rtasida

$2(\bar{a}^2+\bar{b}^2)=|\bar{a}+\bar{b}|^2+|\bar{a}-\bar{b}|^2$ - tenglik o'rinli.

Shu tenglikdan foydalanamiz.

$2(13^2+19^2)=24^2+|\bar{a}-\bar{b}|^2;\ (|\bar{a}|^2=\bar{a}^2);$

$(\bar{a}-\bar{b})^2=1060-576;\quad (\bar{a}-\bar{b})^2=484\Rightarrow|\bar{a}-\bar{b}|=22;$

Javob: $|\bar{a}-\bar{b}|=22;$

2-***usul.*** $|\bar{a}+\bar{b}|^2=(\bar{a}+\bar{b})^2=\bar{a}^2+2\bar{a}\bar{b}+\bar{b}^2$ bundan $24^2=13^2+2\bar{a}\bar{b}+19^2;$

$\bar{a}\bar{b}=576-530;\quad 2\bar{a}\bar{b}=46\Rightarrow\bar{a}\bar{b}=23;\ (\bar{a}-\bar{b})^2=\bar{a}^2-2\bar{a}\bar{b}+\bar{b}^2=169-2\cdot 23+361;$

$(\bar{a}-\bar{b})^2=484\Rightarrow|\bar{a}-\bar{b}|=22;$

Javob: $|\bar{a}-\bar{b}|=22;$

2-Masala $\bar{a}$, $\bar{b}$, $\bar{c}$ - birlik vektorlar uchun $\bar{a}+\bar{b}+\bar{c}=0$ tenglik o'rinli bo'lsa, $\bar{a}\ \bar{b}+\bar{b}\ \bar{c}+\bar{a}\ \bar{c}$ ni hisoblang.

Yechish.

$(\bar{a}+\bar{b}+\bar{c})^2=\bar{a}^2+\bar{b}^2+\bar{c}^2+2\ (\bar{a}\ \bar{b}+\bar{b}\ \bar{c}+\bar{a}\ \bar{c})$

$0=1+1+1+2\ (\bar{a}\ \bar{b}+\bar{b}\ \bar{c}+\bar{a}\ \bar{c})$ bundan $\bar{a}\ \bar{b}+\bar{b}\ \bar{c}+\bar{a}\ \bar{c}=-\frac{3}{2};$

Javob: $\bar{a}\ \bar{b}+\bar{b}\ \bar{c}+\bar{a}\ \bar{c}=-1{,}5.$

3-Masala

$\bar{a}=\{3;-2\}$ $\bar{b}=\{-2;1\}$ $\bar{c}=\{7;-4\}$ vektorlar berilgan. Har bir vektorni qolgan ikki bazis vector orqali ifodalang.

Yechish. $\bar{a}=\alpha\cdot\bar{b}+\beta\cdot\bar{c}$ ifodadan α,β – noma'lumlarni toping. $\begin{cases}-2\cdot\alpha+7\cdot\beta=3\\ \alpha-4\cdot\beta=-2\end{cases}$

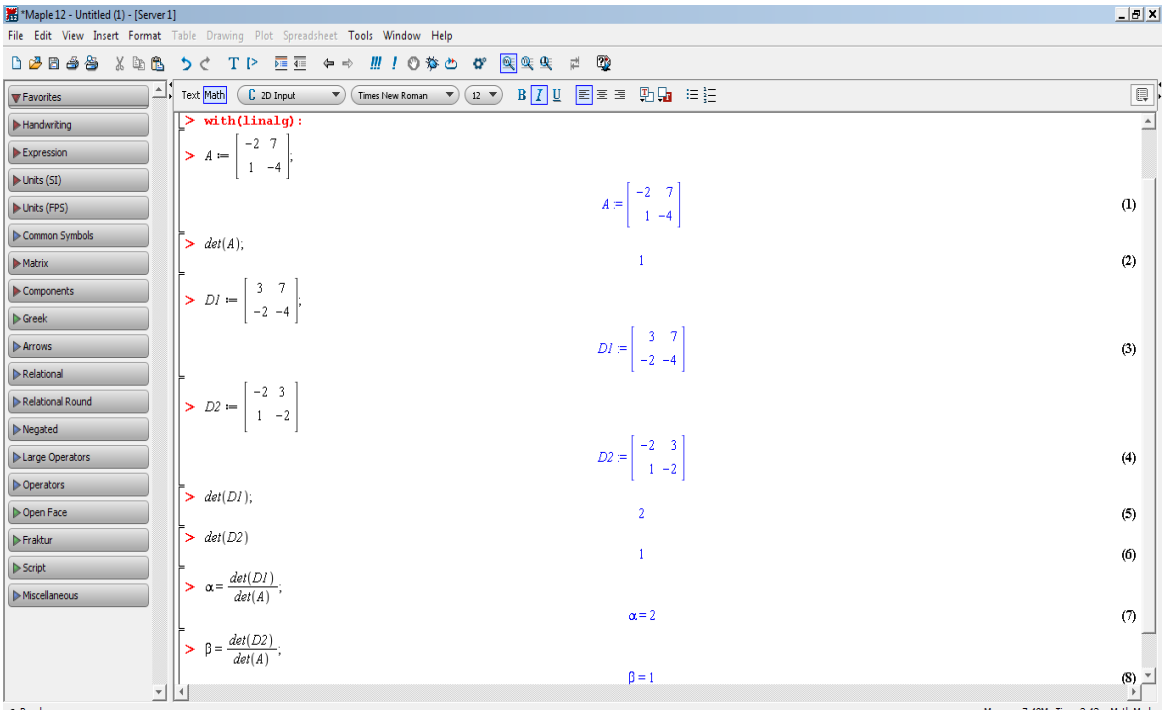
sistemani Kramer formulasi orqali yechamiz. $\Delta=\begin{vmatrix}-2 & 7\\ 1 & -4\end{vmatrix}=8-7=1;$

$$\Delta_\alpha=\begin{vmatrix}3 & 7\\ -2 & -4\end{vmatrix}=-12+14=2;\ \Delta_\beta=\begin{vmatrix}-2 & 3\\ 1 & -2\end{vmatrix}=4-3=1;$$

$\alpha=\dfrac{\Delta_\alpha}{\Delta}=\dfrac{2}{1}=2;\quad \beta=\dfrac{\Delta_\beta}{\Delta}=1;$ $\bar{b}$ va $\bar{c}$ vektorlar ham shu usulda topiladi.

Javob: $\bar{a}=2\bar{b}+\bar{c}$; $\bar{b}=\dfrac{1}{2}\bar{a}-\dfrac{1}{2}\bar{c}$; $\bar{c}=\bar{a}-2\bar{b}$;

Bu masalani Maple paketi yordamida bajarsak quyidagicha natijalar olish mumkin:



Olingan natijalardan ko'rinib turibdiki, masalani ana'naviy klassik usulda echishda va Maple paketi yordamida echishda, aniqlangan α,β – noma'lumlarni qiymatlari bir xil.

Masalani, Maple paketi yordamida echish ancha qulay, ya'ni masala AKT usillari yordamida yechilganda vaqt tejaladi, bu esa ko'roq masala echish imkonini beradi.

4-Masala $\bar{a} = \alpha i - 3j + 2k$ va $\bar{b} = i + 2j - \alpha k$ vektorlar o'zaro perpendikulyar bo'lsa, α ni toping.

Yechish. $\bar{a}\,\bar{b} = (\alpha i - 3j + 2k)(i + 2j - \alpha k) = \alpha i^2 + 2\,\alpha ij - \alpha^2 ik - 3ij - 6j^2 - 3\,\alpha jk -$ $-2ik + 4jk + 4jk - 2\,\alpha k^2 = 0$; $i^2 = j^2 = k^2 = 1$ hamda i; j va k vektorlar perpendikulyar bo'lganligi uchun ularning o'zaro skalyar ko'payitmasi 0 ga teng. Bundan esa, $\alpha - 6 - 2\,\alpha = 0$; $\alpha = -6$; ***Javob***: $\alpha = -6$.

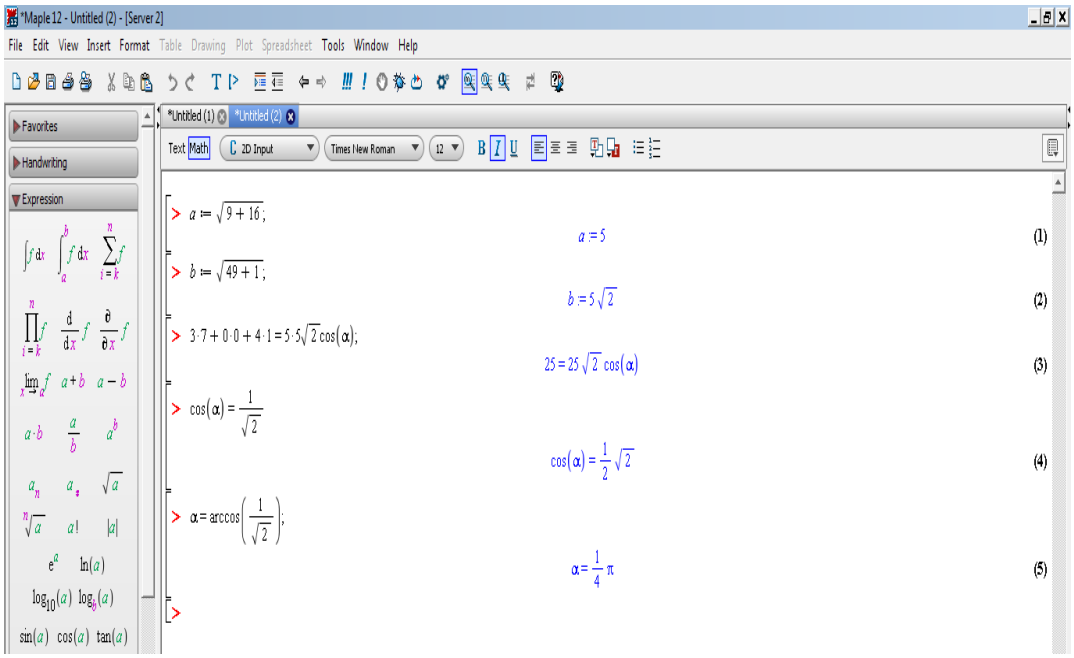
5-Masala

A(-1;-2;4) , B(-4;-2;0) va C(3;-2;1) nuqtalar uchburchakning uchlari. B uchdagi ichki burchakni toping.

Yechish. $\overline{BA} = \{3;0;4\}$, $\overline{BC} = \{7;0;1\}$ $\overline{AB} \cdot \overline{BC} = \left|\overline{AB}\right| \cdot \left|\overline{BC}\right| \cdot \cos\varphi$; bunda α $\overline{AB}$ va $\overline{BC}$ orasidagi burchak. $\left|\overline{AB}\right| = \sqrt{9+16} = 5$; $\left|\overline{BC}\right| = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$;

$3 \cdot 7 + 0 \cdot 0 + 4 \cdot 1 = 5 \cdot 5\sqrt{2} \cdot \cos\alpha$; $25 = 25\sqrt{2} \cdot \cos\alpha$; $\cos\alpha = \frac{1}{\sqrt{2}}$; $\alpha = 45^0$; ***Javob:*** $\alpha = 45^0$.

Bu masalani ham Maple paketi yordamida bajarsak quyidagicha natijalar olish mumkin:



Olingan natijalarni solishtirib, masalani ana'naviy klassik usulda echishda va Maple paketi yordamida yechishda bir xil natija olinganiga guvoh bo'lamiz.

MUSTAQIL ECHISH UCHUN MISOLLAR

№ 1. $\vec{a}=\{3;2;-5\}$ va $\vec{b}=\{2;-3;1\}$ vektorlar berilgan. Quyidagilarni toping:

a) $2\bar{a}-\bar{b}$ vektorlarning koordinata o'qlaridagi proyektsiyalarini; **b)** $2\bar{a}-\bar{b}$ vektorning uzunligini; **c)** $2\bar{a}-\bar{b}$ vektorning yo'naltiruvchi kosinuslarini toping.

№ 2. $\bar{a}=\{6;3;-2\}$ vektorning modulini toping.

№ 3. A (2;-1;0) va V (-1;0;2) nuqtalar berilgan. $\overline{AB}$ va $\overline{BA}$ larning koordinatalarini toping.

№ 4. Tekislikda A (0;-2), B (4;2) va C (4;-2) nuqtalar berilgan. Koordinatalar boshidan $\overline{OA}$, $\overline{OB}$ va $\overline{OC}$ kuchlar qo'yilgan. Ularning teng ta'sir etuvchisi $\overline{OM}$ yasalsin va uning o'qlardagi proyektsiyalari hamda uzunligi topilsin.

$\overline{OA},\overline{OB},\overline{OC}$ va $\overline{OM}$ kuchlar i va j birlik vektorlar orqali ifodalansin.

№ 5. Uchta komplanar m, n va r birlik vektor berilgan bo'lib, (m^ n)=30° va (n^ p)=60° u=m+2n-3p vektor yasalsin va uning moduli hisoblansin.

№ 6. A (3;-1;2) va V (-1;2;1) nuqtalar berilgan. $\overline{AB}$ va $\overline{BA}$ larning koordinatalarini toping.

№ 7. M (5;4;-3) nuqta yasalsin va uning radius-vektorining uzunligi hamda yo'nalishi aniqlansin.

№ 8. $\bar{a}=\{3;2;7\}$ $\bar{b}=\{-1;2;1\}$ vektorlarning yig'indisini va ayirmasini toping.

№ 9. $\bar{r}(0;2;-3)$ radius vektorning ortlar bo'yicha yoyilmasini yozing va modulini hisoblang.

№ 10. Vektor Ox va Oz o'qlar bilan mos ravishda 40° va 80° burchak tashkil etadi. Bu vektorning Oy o'q bilan tashkil etgan burchagi topilsin.

№ 11. A (2;1;-1) nuqtaga R=7 kuch qo'yilgan. Bu kuchning ikki koordinatasi x=2 va y=-3 O'sha kuchni ifodalovchi vektorning yo'nalishi va oxirgi nuqtasi aniqlansin.

№ 12. A (2;2;0) va V (0;-2;5) nuqtalar berilgan. $\overline{AB}=u$ vektor yasalsin, hamda uning uzunligi va yo'nalishi aniqlansin.

№ 13. $\bar{a}$ =-i+j va $\bar{b}=i-2$j+2k vektorlar orasidagi burchak aniqlansin.

№ 14. $\bar{a}$ =i+j+2k va $\bar{b}=i-$j+4k vektorlar berilgan. pr$_v a$ va pr$_a v$ aniqlansin.

№ 15. Agar m va n o'zaro 30^0 burchak tashkil etuvchi birgalik vektorlar bo'lsa, 1) $(m+n)^2$; 2) agar $a=2\sqrt{2}$ va b=4 hamda $(ab)=135^0$ bo'lsa, $(a-b)^2$ hisoblansin.

№ 16. $\bar{a}=\{8;-5;3\}$ va $\bar{b}=\{-4;1;-1\}$ vektorlarga qurilgan parallelogramm

dioganallari uzunliklarini toping.

№ 17. $\bar{a}=\{3;4;-12\}$ vektorning ortini toping.

№ 18. Agar $|\bar{a}|=2,\ |\bar{b}|=3$ bo'lib, ular o'zaro 60^0 li burchak tashkil etsa, $2\bar{a}-\bar{b}$ va $2\bar{a}+3\bar{b}$ vektorlarning skalyar ko'paytmasini toping.

№ 19. Agar $|\bar{a}|=3, |\bar{b}|=4$ bo'lib, $\bar{a}$ va $\bar{b}$ vektorlar orasidagi burchak $\alpha=\frac{2}{3}\pi$ bo'lsa, quyidagilarni hisoblang:

a) $\bar{a}\cdot\bar{b};$ b) $\bar{a}^2;$ v) $\bar{b}^2;$ g) $(\bar{a}+\bar{b})^2;$ d) $(\bar{a}-\bar{b})^2;$ ye) $(3\bar{a}-2\bar{b})\cdot(\bar{a}+2\bar{b})$

№ 20. $\bar{a}=\{m;3;4\}$ va $\bar{b}=\{4;m;-7\}$ vektorlar berilgan. m ning qanday qiymatida bu vektorlar perpendikulyar bo'ladi?

№ 21. $\bar{F}=\{3;-2;-5\}$ kuchning qo'yilish nuqtasini to'g'ri chiziq bo'ylab harakat qilib, $M_1(2;-3;5)$ holatdan $M_2(3;-2;-1)$ holatga o'tadi. Bu ko'chishda F kuch bajargan ishni hisoblang.

№ 22. To'rtburchakning uchlari berilgan: A (1;-2;2) , B (1;4;0) , C (-4;1;1) , D (-5;-5;3) .

SHu to'rtburchakning AC va BD dioganallari o'zaro perpendikulyar bo'lishini isbotlang.

№ 23. A (-2;3;-4) , B (3;2;5) , C (1;-1;2) , D (3;2;-4) nuqtalar berilgan. $\overline{AB}$ vektorning $\overline{CD}$ vektordagi proyektsiyasini hisoblang.

№ 24. Uchburchakning uchlari berilgan: A (1;2;1) , B(3;-1;7) , S (7;4;-2) uning ichki burchaklarini toping.

№ 25. A (a;0;0) , B (0;0;2a) , C (a;0;a) nuqtalar berilgan. $\overline{OC}$ va $\overline{AB}$ vektorlar yasalsin va ular orasidagi burchak topilsin.

№ 26. $\bar{a}=\{3;-2;6\}$ va $\bar{b}=\{-2;1;0\}$ berilgan. Quyidagi vektorlarning koordinata o'qlaridagi proyektsiyalarini toping. 1) $\bar{a}+\bar{b}$; 2) $\bar{a}-\bar{b}$; 3) $2a$; 4) $-\frac{1}{2}b$; 5) $2a+3b$; 6) $\frac{1}{3}a-b$; 7) $3b$; 8) $\frac{1}{3}a$

№ 27. Ikkita $\bar{a}=\{-1;2;3\}$ va $\bar{b}=\{2;-4;5\}$ vektor berilgan. Quyidagi vektorlarning koordinata o'qidagi proyektsiyalarini toping.

a) $2\bar{a}+\bar{b}$; b) $\bar{a}-3\bar{b}$; c) $3\bar{a}+5\bar{b}$.

№ 28. $\bar{a}=\{2;3;6\}$ vektorlarning yo'naltiruvchi kosinuslarini toping.

№ 29. Quyidagi vektorlar uzunliklari berilgan: $|\bar{a}|=13$

$|\bar{b}|=19 \quad |\bar{a}+\bar{b}|=24. \; |\bar{a}-\bar{b}|$ ni hisoblang.

№ 30. A(1,2,3) va B(3,-4,6) nuqtalari berilgan. $\overline{AB}$ vektor uzunligini toping va yo'nalishi toping.

№ 31. $\bar{a}=\{3;4;-12\}$ vektor ortini toping.

№ 32. α va β burchaklarning qanday qiymatlarida $\bar{a}=-2i+3j+\beta k$ va $\bar{b}=1i-6j+2k$ vektorlar kollinear bo'ladi.

№ 33. α va β burchaklarning qiymatlarida $\bar{a}=2\bar{i}+2\bar{j}+\bar{k}$ va $\bar{b}=3\bar{i}-6\bar{j}+\beta\bar{k}$ vektorlar kollinear bo'ladi.

№ 34. Vektorlar ortini toping:

1) $\bar{a}=\{6;-2;-3\}$ 2) $\bar{a}=\{3;4;-12\}$.

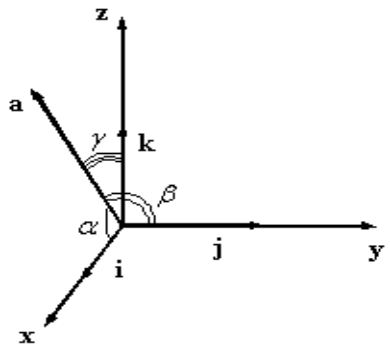
4 - §. Vektorning yo'naltiruvchi kosinuslari. Vektorning koordinatalari va koordinatalari bilan berilgan vektorlar ustida amallar

1.4.1 Yo`naltiruvchi kosinuslar

Ta'rif Vektorni koordinata o'qlari musbat yo'nalishlari bilan hosil qilgan burchsklari kosinuslari, *vektorning yo'naltiruvchi kosinuslari* deyiladi.

a vektor koordinata o'qlarining musbat yo'nalishlari bilan mos ravishda α, β, γ burchaklar tashkil qilsin. $\cos\alpha$, $\cos\beta$, $\cos\gamma$ lar **a** vektorning yo'naltiruvchi kosinuslari deyiladi. $|\mathbf{a}|$ va bu kosinuslar berilgan bo'lsin. $X=|\mathbf{a}|\cdot\cos\alpha$, $Y=|\mathbf{a}|\cdot\cos\beta$, $Z=|a|\cdot\cos\gamma$.

Demak $\cos\alpha=\frac{X}{\sqrt{x^2+y^2+z^2}},\cos\beta=\frac{Y}{\sqrt{x^2+y^2+z^2}},\cos\gamma=\frac{Z}{\sqrt{x^2+y^2+z^2}}$ Bundan esa $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ ekani kelib chiqadi.



16 - chizma

Teorema Vektorning koordinatalari, *vektorning yo'naltiruvchi kosinuslarini*, vector uzunligiga ko'paytmasiga teng.

Natija Agar vector birlik vector bo'lsa, uning koordinatalari *vektorning yo'naltiruvchi kosi nuslariga* teng.

1- **Masala** $\bar{a}=\{12;-15;-16\}$ vektorni yo'naltiruvchi kosinusini toping.

Yechish. $\bar{a}=\{a_1;a_2;a_3\}$ bo'lsa, u holda $\cos\alpha=\frac{a_1}{|\bar{a}|}$; $\cos\beta=\frac{a_2}{|\bar{a}|}$; $\cos\gamma=\frac{a_3}{|\bar{a}|}$;

$$\cos\alpha=\frac{12}{\sqrt{12^2+(15)^2+(-16)^2}}=\frac{12}{\sqrt{625}}=\frac{12}{25};\ \cos\beta=\frac{-15}{25}=-\frac{3}{5};\ \cos\gamma=-\frac{16}{25};$$

2- **Masala** Vector koordinata o'qlari bilan quyidagi burchaklarni tashkil etishi mumkinmi? 1) $\alpha=45^0$; $\beta=60^0$; $\gamma=120^0$; 2) $\alpha=45^0$; $\beta=135^0$;$\gamma=60^0$;

Yechish. Vektorning yo'naltiruvchi koordinatalari uchun $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ tenglik o'rinli. Masalani yechishda shu tenglikdan foydalanamiz.

$$\cos^2 45^0+\cos^2 60^0+\cos^2 120^0=1;\ \left(\frac{\sqrt{2}}{2}\right)^2+\left(\frac{1}{2}\right)^2+\left(-\frac{1}{2}\right)^2=1;\ \frac{2}{4}+\frac{1}{4}+\frac{1}{4}=1\Rightarrow 1=1;$$

tenglik o'rinli. Demak, (→) koordinata o'qlari bilan $\alpha=45^0$; $\beta=60^0$; $\gamma=120^0$; burchakni tashkil qiladi.

2) $cos^2\,4\,5^0+cos^2\,1\,35^0+cos^2\,6\,0^0=1;;$ $\frac{2}{4}+\frac{2}{4}+\frac{1}{4}=1\Rightarrow\frac{5}{4}\neq 1$ tenglik to'g'ri emas.

Ushbu vektor koordinata o'qlari bilan mos ravishda $\alpha=45^0$; $\beta=135^0$; $\gamma=60^0$; burchaklar tashkil etar ekan.

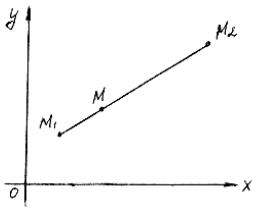
1.4.2 Ikki nuqta orasidagi masofa.
Kesmani berilgan nisbatda bo'lish

1. Faraz qilaylik tekislikda M_1 (x_1, y_1), M_2 (x_2, y_2) nuqtalar berilgan bo'lsin. U holda $\overline{M_1M_2}$ vector $\overline{M_1M_2}=\{x_2-x_1,y_2-y_1\}$ koordinatalarga ega bo'ladi va uning uzunligi$d=|\overline{M_1M_2}|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$kabi aniqlanadi.

2. Fazoda $M_1(x_1, y_1, z_1)$, $M_2(x_2, y_2, z_2)$ nuqtalar berilgan bo'lsin. U holda $\overline{M_1M_2}$ vector $\overline{M_1M_2} = \{x_2 - x_1, y_2 - y_1, z_2 - z_1\}$ koordinatalarga ega bo'ladi va uning uzunligi $d = \left|\overline{M_1M_2}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ kabi aniqlanadi.

3. M_1 va M_2 nuqtalar tekislida berilgan bo'lsin. M_1M_2 to'g'ri chiziqda yotuvchi ixtiyoriy M(x,y) nuqtani shunday topish kerakki $\frac{\left|\overline{M_1M}\right|}{\left|\overline{M_2M}\right|} = \lambda$ tenglik o'rinli bo'lsin, bunda $\lambda \neq 0$ berilgan son. Demak, $\frac{x - x_1}{x_2 - x} = \lambda$ dan $x = \frac{x_1 + \lambda \cdot x_2}{1 + \lambda}$ ekanligi kelib chiqadi.

Shuning kabi $y = \frac{y_1 + \lambda \cdot y_2}{1 + \lambda}$ ni ham aniqlash mumkin. Agar $\lambda=1$ bo'lsa M_1M_2 kesma o'rtasining koordinatalari hosil bo'ladi: $x = \frac{x_1+x_2}{2}, y = \frac{y_1+y_2}{2}$



17 - chizma

4. M_1 va M_2 nuqtalar fazoda berilgan bo'lsin. M_1M_2 to'g'ri chiziqda yotuvchi ixtiyoriy M(x,y,z) nuqtani shunday topish kerakki $\frac{\left|\overline{M_1M}\right|}{\left|\overline{M_2M}\right|} = \lambda$ tenglik o'rinli bo'lsin, bunda $\lambda \neq 0$ berilgan son. Demak, $\frac{x - x_1}{x_2 - x} = \lambda$ dan $x = \frac{x_1 + \lambda \cdot x_2}{1 + \lambda}$ ekanligi kelib chiqadi.

Shuning kabi $y = \frac{y_1 + \lambda \cdot y_2}{1 + \lambda}$ va $z = \frac{z_1 + \lambda \cdot z_2}{1 + \lambda}$ larni ham aniqlash mumkin. Agar $\lambda=1$ bo'lsa M_1M_2 kesma o'rtasining koordinatalari hosil bo'ladi: $x = \frac{x_1+x_2}{2}, y = \frac{y_1+y_2}{2}, z = \frac{z_1+z_2}{2}$

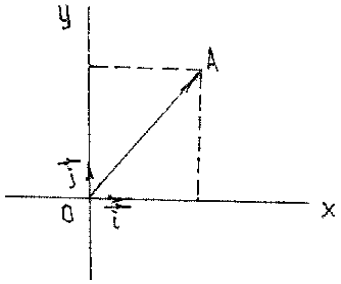
1.4.3. Vektorlarni qo`shishning umumiy qoidasi

$\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}, \ldots, \mathbf{a_n}$ vektorlar berilgan bulsa ularning yigindisini topish uchun shaklda ko`rsatilgan kabi tartibda ularni joylashtirsak birinchi vektor boshiga oxirgi vektor oxiri bilan tutashtiruvchi vektor bu vektorlar yig`indisi bo`ladi. **a** va **b** vektorlar, λ va μ sonlar bo`lsin. U holda;

1. $(\lambda+\mu)\cdot\mathbf{a}=\lambda\mathbf{a}+\mu\mathbf{a}$;
2. $\lambda\cdot(\mu\mathbf{a})=(\lambda\mu)\mathbf{a}$;
3. $\lambda(\mathbf{a}+\mathbf{b})=\lambda\mathbf{a}+\lambda\mathbf{b}$ bo'ladi.

1.4.4. Vektorlarni fazoda koordinata bazisi orqali tashkil yetuvchilarga ajratish

Ta'rif 1. Musbat yo'nalishlari mos ravishda $\vec{\imath},\vec{\jmath}$ vektorlar bilan aniqlanuvchi o'zaro perpendikulyar ikkita o'qdan tashkil topgan sistema tekislikda to'g'ri burchakli koordinatalar sistemasi deyiladi va $R=\{O,\vec{\imath},\vec{\jmath}\}$ ko'rinishda belgilanadi. O nuqta koordinatalar boshi, $\vec{\imath},\vec{\jmath}$ birlik vektorlar esa koordinata vektorlari deyiladi. Ta'rifga asosan, $\vec{\imath},\vec{\jmath}$ vektorlar ortogonal va birlik vektorlardir: $|\vec{\imath}| = |\vec{\jmath}| = 1$; $\vec{\imath}\perp\vec{\jmath}$. Musbat yo'nalishlari $\vec{\imath},\vec{\jmath}$ vektorlar bilan aniqlangan o'qlar mos ravishda abssissalar va ordinatalar o'qlari deb ataladi.



18 – chizma

Tekislikda har qanday holdan farqli vektorni bazis vektorlar orqali $\mathbf{a}=OA_x+OA_y=\lambda\mathbf{i}+\mu\mathbf{j}$ ko'rinishda yoyish mumkin.

Bu erda **i, j** lar koordinata bazisi deyiladi. $OA_x=\lambda=\Pi p_x\mathbf{a}=X$, $OA_y=\mu=\Pi p_y\mathbf{a}=Y$, Demak $\mathbf{a}=X\mathbf{i}+Y\mathbf{j}$

Ta'rif 2. Agar **i, j, k** vektorlar quyidagi shartlarni qanoatlantirsa:

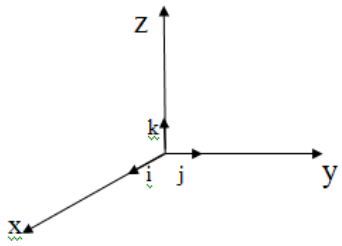
1) ular mos ravishda OX, OY, OZ o'qlari musbat yo'nalishida yotadi, yani bu vektorlarning har biri o'z o'qida musbat tomonga yo'nalgan;

2) **i, j, k** lar birlik vektorlar, ya'ni $|i|=|j|=|k|=1$;

3) ular o'zaro perpendikulyar bo'lsa, bu vektorlarga bazis vektorlar deyiladi.

Fazoda har qanday holdan farqli vektorni bazis vektorlar orqali $\mathbf{a}=OA_x+OA_y+OA_z=\lambda\mathbf{i}+\mu\mathbf{j}+\nu\mathbf{k}$ ko'rinishda yoyish mumkin.

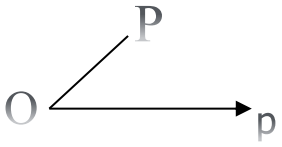
Bu erda **i, j, k** lar koordinata bazisi deyiladi. $OA_x=\lambda=\Pi p_x\mathbf{a}=X$, $OA_y=\mu=\Pi p_y\mathbf{a}=Y$, $OA_z=\nu=\Pi p_z\mathbf{a}=Z$. Demak $\mathbf{a}=X\mathbf{i}+Y\mathbf{j}+Z\mathbf{k}$



19 - chizma

1.4.5. Qutb koordinatalar sistemasi

Ba'zi bir amaliy masalalarni yechishda Dekart sistemasidan foydalanish noqulay, shuning uchun yangi qutb koordinatalar sistemasini kiritish maqsadga muvifiq bo'ladi. Qutb koordinatalar sistemasini quyidagicha kiritamiz. Tekislida ixtiyoriy nqta (O) olib uni qutb boshi deb qabul qilamiz, qutb o'qi esa O niqtadan chiqib o'nnga yo'nalgan bo'lsin.

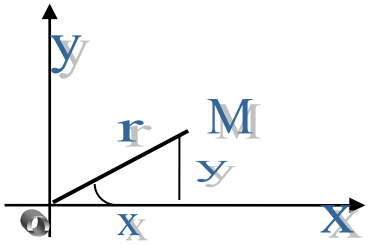


20-chizma

Dekart va qutb koordinatalar sistemalari orasidagi bo'g'liqlikni aniqlash uchun, Dekart sistemasini shunday tanlaymizki, koordinatalar boshi qutb o'qi, Ox o'qining"musbat" yo'nalishi qutib yo'nalishi bilan mos tushsin.

Faraz qilaylik, M nuqta Dekart koordinatalar sistemasida M(x,y), qutb koordinatalar sistemasida M(φ,ρ) koordinatalarga ega bo'lsin. U holda bu sistemalar orasidagi bog'liqliklar quyidagicha aniqlanadi

$$\begin{cases} x=\rho\cos\alpha \\ y=\rho\sin\alpha \end{cases} \text{ va } \rho=\sqrt{x^2+y^2}, \qquad \alpha=\begin{cases} arctg\frac{y}{x} & agar\ x\succ 0\ bo'lsa \\ \pi+arctg\frac{y}{x} & agar\ x\prec 0\ bo'lsa \end{cases}$$



21-chizma

1.4.6 Vektorlarnini geometrik masalalarni yechishga tatbiqi

Qo`yida vektorlarni qo`llab yechishga oid ba`zi masalalarni qarab chiqaylik.

Ma`lumki, vektorlarni qo`llab maktab geometriya va analitik geometriya kurslarida uchraydegan ko`pgina masalalarni osonlikcha yechish mumkun. Hisoblashga oid masalalarni vektorlarni qo`llab yechish, konstrutni yondoshishga ya`ni qo`shimcha yasash hamda elementar algebra va trigonometriya aparatlarni qo`llashga nisbatan ancha qulay.

Vektorlar yordamida geometrik masalalarni samarali yechishda, nafaqat vektor algebrasi qonunlarini qullay bilish, balki geometrik masalani vektor "tili"ga o`girish va masala yechish usulini to`g`ri tanlay olish hamda uni yechish rejasini to`g`ri tuzush talab etiladi.

Shuning uchun, ba`zi bir asosiy holatlarini ko`rsatib o`tish kerak deb hisoblaymiz.

1. Agar masofa yoki burchakni hisoblash talab etilsa, u holda ikki vektor skalyar ko`paytmasini qo`llash samarali natija beradi.

2. Geometrik masalalarni yechish uchun vektorlarni qo`llashda ikki xil yo`l tutish kerak bo`ladi:

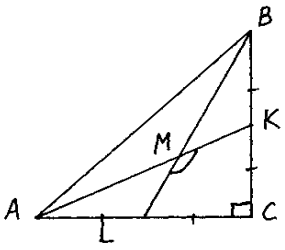
a) berilgan vektorlarni qo`yish uchun boshlang`ich nuqtani to`g`ri tanlash;

b) qaralayotgan masaladagi chizmaga qarab, vektorlarni yo`naltirilgan kesmalar yordamida tasvirlash va bunda ularni bir nuqtadan boshlab qo`ymaslik;

3. Agar masala planametriyaga oid bo`lsa, u holda ikkita kollinar bo`lmagan vektorlarni ajratib ularni bazns vektorlar deb hisoblab, qolgan vektorlarni ular orqali ifodalash kerak, agar qaraliyotgan masala stereometriyaga ta`luqli bo`lsa, bazis vektorlar sifatida uchta nokomplanar vektorlar olinadi. Bu hollarda vektorning boshlang`ich nuqtasi kiritish-tanlash shart emas.

4 Ko`pgina hollarda, masalan, ko`p yoqli ko'pburchaklarga oid masalalar echishga, ko`pyoqli burchak uchidan chiquvchi birlik vektorlar kiritilsa hisoblash Ishlariy engillashadi uchburchak o`tkir burchakli uchlaridan chiquvchi medinalarning kesishishdan hosil bo`lgan o'tmas burchakni hisoblang. (1 chizma)

1 - masala. To`g`ri burchakli teng yonli uchburchak o`tkir burchakli uchburchak uchlaridan chiquvchi mednanalarning kesishidan hosil bo`lgan o`tmas burchakni hisoblang . (1 chizma)



1-chizma

Yechish. $\overrightarrow{CA} = \vec{a}$va $\overrightarrow{CB} = \vec{b}$bo`lsin.Shartga asosan $|\vec{a}| = |\vec{b}|$ bo`ladi. $\overrightarrow{AK}$ vektor$\overrightarrow{CK}$ va $\overrightarrow{CA}$ vektorlar ayirmasiga teng ya`ni, $-\overrightarrow{CA} = \frac{1}{2}\vec{b} - \vec{a}$ (chunki. $\overrightarrow{CK} = \frac{1}{2}\overrightarrow{CB}$). Shunga o`xshash $\overrightarrow{BL} = \frac{1}{2}\overrightarrow{CA} - \overrightarrow{CB} = \frac{1}{2}\vec{a} - \vec{b}$ bo`ladi. Ma`lumki, ikki vektor orasidagi burchak $cos\,\alpha = \frac{(\overrightarrow{AK}\cdot\overrightarrow{BK})}{|\overrightarrow{AK}|\cdot|\overrightarrow{BL}|}$ kabi topiladi, lekin,

1. $\left(\overrightarrow{AK}\cdot\overrightarrow{BL}\right)=\left(\frac{1}{2}\vec{b}-\vec{a}\right)\cdot\left(\frac{1}{2}\vec{a}-\vec{b}\right)=\frac{1}{4}|\vec{a}|\cdot|\vec{b}|\,cos\quad(\vec{a}\vec{b})-\frac{1}{2}|\vec{b}|^2-\frac{1}{2}|\vec{a}|^2+|\vec{a}|\cdot|\vec{b}|\,cos(\vec{a}\vec{b})$bo`lgani uchun

$\left(\overrightarrow{AK}\cdot\overrightarrow{BL}\right)=-\frac{1}{2}|\vec{b}|^2-\frac{1}{2}|\vec{a}|^2=-\frac{1}{2}|\vec{a}|^2-\frac{1}{2}|\vec{a}|^2=-|\vec{a}|^2$bo`ladi, chunki $\vec{a}\perp\vec{b}$ bo`lgani uchun $cos\quad(\vec{a}\vec{b})=\vec{o}$ bo`ladi. $\overrightarrow{AK}$ va $\overrightarrow{BL}$ vektorlar uzunligi Pifagor teorimasiga asosan hisoblanadi. $\overrightarrow{BL}=\overrightarrow{AK}=\sqrt{AC^2+CK^2}=\sqrt{|a|^2+\frac{1}{4}|a|^2}=\frac{1}{2}|a|\sqrt{5}$ Shunday qilib $cos\,\alpha=\frac{-|a|^2}{\frac{1}{4}|a|^2\cdot 5}=-\frac{4}{5}$ U holda $\alpha=\text{ar}\,cos\left(-\frac{4}{5}\right)$ ni topamiz

2 - masala

A (2; -1; 3) nuqtaga $\vec{F}$ (3; 4; −2) kuch qo`yilgan. Bu kuchning koordinatalar boshiga nisbatan momenti va koordinata o`qlari bilan hosil qilgan burchaklarini toping.

Yechish: A nuqtaga qo`yilgan kuchning koordinatalar boshiga nisbatan momenti A nuqtaning radus-vektorlari $\vec{r}$bilan $\vec{F}$kuchning vektor ko`paytmasiga teng, ya`ni$m_0(F)=\left[\vec{r}\cdot\vec{F}\right]$. A nuqtaning radus-vektorlarning koordinata o`qlarining proeksiyalari: r_x=x=2; r_y=y=-1; r_z=z=3 yoki $\vec{r}=2\,\vec{i}-\vec{j}+3\,\vec{k}$. F qo`shni birlik vektorlar orqali ifodalasak, $\vec{F}=3\,\vec{i}+4\,\vec{j}-2\,\vec{k}$ $\vec{F}$ kuchning

koordinatalari boshiga niabatan momenti: $m_0(F) = \left[\overrightarrow{r} \cdot \overrightarrow{F}\right] = -10\,\overrightarrow{i} + 13\,\overrightarrow{j} + 11\,\overrightarrow{k}$ kabi aniqlanadi. Bundan m_x=-10; m_y=13; m_z=11 bo`lgani uchun, $|m_0(F)| = \sqrt{100 + 169 + 121} = \sqrt{390}$ekanini topamiz. Kuch momentining yo`naltiruvchi kosinuslari esa $Cos\alpha = -\frac{10}{\sqrt{390}}; Cos\beta = \frac{13}{\sqrt{390}}; Cos\gamma = -\frac{11}{\sqrt{390}}$ ga teng bo`ladi. Bulardan esa berilgan kuchnung koordinata o`qlari bilan hosil qilgan burchaklarni aniqlaymiz $\alpha \approx 120^0 24'; \beta \approx 48^0 51'; \gamma \approx 56^0 9'$.

VII bob
SKALYAR KO`PAYTMA VA UNING XOSSALARI

1- §. Ikki vektorning skalyar ko`paytmasi. Skalyar ko`paytma xossalari

Ta'rif: Ikki vektorning skalyar ko'paytmasi deb, ular uzunliklari ko'paytmasining ular orasidagi burchak kosinusi ko'paytmasiga aytiladi.

Ikki vektorning skalyar ko'paytmasi **ab** deb belgilanadi va **a**·**b**=|**a**|·|**b**|·cosφ kabi aniqlanadi, φ=(**a,b**) |**b**|·cosφ=Пр$_a$**b**, **ab**=|**a**|·Пр$_a$**b**=|**b**|·Пр$_b$**a** ekani ravshan.

Skalyar ko`paytma tushunchasining manbai mexanikada. Haqiqatan ham agar **a** vektor **b** vektorning boshidan ohiriga qarab sirg`alib ta'sir etuvchi kuch bo`lsa, u vaqtda bu kuch bajargan W ish W=|**a**|·|**b**|·cosφ formula bilan aniqlanadi.

Skalyar ko`paytmaning algebraik xossalari:

1) **ba**=$\boldsymbol{ab} = |\boldsymbol{a}|\text{пр}_{\boldsymbol{a}}\boldsymbol{b} = |b|\text{пр}_b a$ agar $\boldsymbol{a} \neq 0$ skalyar ko'paytmaning kommutativligi

2) (α **a**)**b**= α (**ab**) (demak bundan (α **a**)(β **b**)= αβ (**ab**) ekanligi kelib chiqadi)

3) **a**(**b**+**c**)=**ab**+**ac**

Eslatma. Uchta va undan ortiq vektorlarning skalyar ko'paytmasi ma'noga ega emas (chunki **ab**-son va (**ab**)***c** - endi vektordir).

Skalyar ko'paytmaning geometrik hossalari.

4) agar **a** va **b** o'tkir burchak tashkil qilsa **ab**>0

5) bu burchak o'tmas bo'lsa **ab**<0 bo'ladi (**ab**=0 agar! (**a** || **b**)

6) **a**·**a**=|**a**|^2 yoki **a**^{2}=|**a**|^2

7) $\boldsymbol{a}^2 > 0$ *agar* $\boldsymbol{a} \neq 0$ *bo'lsa;*

8) $\boldsymbol{ab} = 0$ *faqat va faqat agar* **a** *va* **b** *vektorlar perpendikulyar bo'lsa*

9) *agar* φ - **a** *va* **b** *vektorlar orasidagi burchak bo'lsa, u holda* $cos\,\varphi = \frac{\boldsymbol{ab}}{|\boldsymbol{a}||\boldsymbol{b}|}$

10) kolleniar vektorlarning skalyar ko'paytmasi, ular modullarining ko'paytmasiga teng, ya'ni $(\vec{a} \cdot \vec{b}) = |\vec{a}| \cdot |\vec{b}|$, chunki kolleniar vektorlar orasidagi burchak $\phi = 0$ hamda $cos\,\phi = 1$.

11) Vektorli ko'phadlarni skalyar ko'paytirishda, skalyar ifodalar uchun algebraning ko'paytirish qoidasi saqlanadi, masalan $(\vec{a} \pm \vec{b})^2 = \vec{a}^{\,2} \pm 2(\vec{a} \cdot \vec{b}) + \vec{b}^{\,2}$.

Masala. Uchburchakning uchlari $A(2;-1;3)$, $B(1;1;1)$, $C(0;0;5)$ nuqtalarda yotsin. Uchburchak AB tomoni zunligi va A uchidagi burchakni toping.

Echish. $\overrightarrow{BA} = (1;-2;2)$, $\overrightarrow{BC} = (-1;-1;4)$, $AB = |\overrightarrow{AB}| = \sqrt{1^2 + (-2)^2 + 2^2} = 3$.
$cos\angle ABC = \frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{|\overrightarrow{BA}|\cdot|\overrightarrow{BC}|}$, $\overrightarrow{BA} \cdot \overrightarrow{BC} = 1 \cdot (-1) + (-2)(-1) + 2 \cdot 4 = 9$,
$\overrightarrow{BA} \cdot \overrightarrow{BC} = 1 \cdot (-1) + (-2)(-1) + 2 \cdot 4 = 9$ $cos\angle ABC = \frac{1}{\sqrt{2}}$, $\angle ABC = 45^\circ$.
Javob: $AB = 3$, $\angle ABC = 45^\circ$.

Koordinata shaklidagi skalyar ko'paytma

a={X_1,Y_1,Z_1} ва **b**={X_2,Y_2,Z_2} bo'lsa, u vaqtda **ab**=$X_1 \cdot X_2 + Y_1 \cdot Y_2 + Z_1Z_2$ bo'ladi. Bu formulani isbotida

i^2=1, **ij**=0, **ik**=0 ;

ji=0, **j**^{2}=1, **jk**=0 ;

ki=0, **kj**=0, **k**^{2}=1;

ekanini inobatga olish kerak.

Natija 1. **a**={X_1, Y_1, Z_1}, **b**={X_2, Y_2, Z_2} vektorning perpendikulyar bo'lishi uchun $X_1X_2+Y_1Y_2+Z_1Z_2 = 0$ bo'lishi zarur va etarlidir.

Natija 2. $cos\,\phi = \frac{X_1X_2+Y_1Y_2+Z_1Z_2}{\sqrt{X_1^2+Y_1^2+Z_1^2}\cdot\sqrt{X_2^2+Y_2^2+Z_2^2}}$ - ikki vektor orasidagi burchakni hisoblash formulasidir.

Masala 1. A(4;3;2) , B(2;-1;5) va C(1;4;-1) nuqtalar berilgan ∠BAC=φ-?

Echish. AB={-2;-4;3} , AC ={-3;1;-3}

Demak, $\cos\varphi = \frac{6-4-9}{\sqrt{4+16+9}\cdot\sqrt{9+1+9}} = \frac{-7}{\sqrt{29}\cdot\sqrt{19}} = -\frac{7}{\sqrt{551}}$; ya'ni burchak o'tmasdir.

Masala 2. A(3;2;5) va B(1;-1;2) nuqtalar berilgan. AB vektorning koordinata o'qlari bilan teng burchaklar tashkil qiluvchi **u** o'qidagi proeksiyasini toping.

Echish. AB={-2;-3;-3}.Faraz qilaylik cosα, cosβ, cosγ lar U o'qning yo'naltiruvchi kosinuslari bo'lsin.

Ma'lumki $\cos^2\alpha+\cos^2\beta+\cos^2\gamma = 1$ va cosα=cosβ=cosγ bo'lgani uchun $3\cos^2\alpha=1$, $\cos\alpha = \frac{1}{\sqrt{3}} = \cos\beta = \cos\gamma$.

Bunday holda **u** o'qga ixtiyoriy vektor **a**(X,Y,Z) ning proeksiyalari $\Pi_p S = X\cdot\cos\alpha+Y\cdot\cos\beta+Z\cdot\cos\gamma$ bo'lgani chun $\Pi_{p_u}\overline{AB} = -2\cdot\frac{1}{\sqrt{3}} - 3\cdot\frac{1}{\sqrt{3}} - 3\frac{1}{\sqrt{3}} = -\frac{8}{\sqrt{3}}$ bo'ladi.

MUSTAQIL ECHISH UCHUN MASALALAR

№ **1.** Agar $|\bar{a}| = 2$ $|\bar{b}| = 3$ bo'lib, ular o'zaro 60^0 li burchakni tashkil etsa, $2\bar{a}-\bar{b}$ va $2\bar{a}+3\bar{b}$ vektorlarni skalyar ko'paytmasini toping.

№ **2.** Agar $|\bar{a}| = 3$ $|\bar{b}| = 4$ bo'lib, $\bar{a}$ va $\bar{b}$ vektorlar orasidagi burchak $\alpha = \frac{2\pi}{3}$ bo'lsa, quyidagilarni hisoblang:

a) $\bar{a}\cdot\bar{b}$, b) $\bar{a}^2$, v) $\bar{b}^2$, g) $(\bar{a}+\bar{b})^2$, d) $(\bar{a}-\bar{b})^2$, ye) $(3\bar{a}-2\bar{b})\cdot(\bar{a}+2\bar{b})$.

№ **3.** $\bar{a}$ va vektorlar orasidagi burchak $\alpha = \frac{\pi}{4}$ va $|\bar{a}| = \sqrt{2}$ $|\bar{b}| = 3$ ekanligi ma'lum. $\bar{c} = 2\bar{a}+3\bar{b}$ vektor uzunligini hisoblang.

№ **4.** $\bar{a}$ va vektorlarning orasidagi burchak $\alpha = \frac{\pi}{3}$ ga teng.

$|\bar{a}| = 3$ $|\bar{b}| = 4$ $\bar{c} = 3\bar{a} + 2\bar{b}$ vektorlar uzunligini hisoblang.

№ **5.** $\bar{a}$ va $\bar{b}$ vektorlarning koordinatalari berilgan:

$\bar{a} = 7\bar{i} + 2\bar{j} + 3\bar{k}$ $\bar{b} = 2\bar{i} - 2\bar{j} + 4\bar{k}.$

Bu vektorlarning skalyar ko'paytmasini toping.

№ **6.** Vektorlar berilgan: $\bar{a} = \{4;-2;-4\}$ $\bar{b} = \{6;-3;2\}.$

1) $\bar{a} \cdot \bar{b}$; 2) $\sqrt{\bar{a}^2}$; 3) $\sqrt{\bar{b}^2}$; 4) $(2\bar{a} - 3\bar{b}) \cdot (\bar{a} + 2\bar{b})$; 5) $(\bar{a} + \bar{b})^2$; 6) $(\bar{a} - \bar{b})^2$.

№ **7.** $\bar{a}$ va $\bar{b}$ vektorlar orasidagi burchak $\alpha = \frac{\pi}{6}$. $|\bar{a}| = \sqrt{3}$ $|\bar{b}| = 1.$ ma'lum.

$\bar{p} = \bar{a} + \bar{b}$ $\bar{q} = \bar{a} - \bar{b}$. Vektorlarning uzunligini hisoblang.

№ **8.** $\bar{a} = \alpha\bar{i} - 3\bar{j} + 2\bar{k}$ va $\bar{b} = \bar{i} + 2\bar{j} - \alpha\bar{k}$ vektorlar berilgan. α ning qanday qiymatida vektorlar o'zaro perpendikulyar.

№ **9.** Agar $|\bar{a}| = 7\sqrt{2}$ $|\bar{b}| = 4$ va $(\bar{a}, \bar{b}) = \frac{\pi}{4}$ bo'lsa, $3\bar{a} + \alpha\bar{b}$ va $\bar{a} - 2\bar{b}$ vektorlar α ning qanday qiymatlarida o'zaro perpendikulyar bo'ladi?

№ **10.** $\bar{a}$ va $\bar{b}$ vektorlarining koordinatalari berilgan

$\bar{a} = 7\bar{i} + 4\bar{j} + 3\bar{k}$ $\bar{b} = 2\bar{i} - 2\bar{j} + 5\bar{k}.$ Bu vektorlarning skalyar ko'paytmasini toping.

№ **11.** $\bar{a} = 3\bar{i} + 4\bar{j}$ $\bar{b} = -3\bar{j} + \bar{k}$ $\bar{c} = 2\bar{j} + 5\bar{k}$ vektorlarda parallelepiped yasalgan hamda uning hajmi hisoblansin. Berilgan (*a*; *b*; *c*) vektorlar qaysi bog'lamni tashkil etadi.

№ **12.** $\bar{a}$ va $\bar{b}$ vetorlar o'zaro perpendikulyar bo'lib, $|\bar{a}| = 3$ va $|\bar{b}| = 4$ bo'lsa, quyidagilarni hisoblang:

a) $|(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})|$ b) $|(3\bar{a} - \bar{b}) \times (\bar{a} - 2\bar{b})|$

№ **13.** $\bar{a}$ va $\bar{b}$ vektorlar o'zaro $\alpha = 45^0$ li burchakni tashkil qilib, $|\bar{a}| = |\bar{b}| = 5$ bo'lsa,

$\bar{p} = \bar{a} - 2\bar{b}$ va $q = 3\bar{a} + 2\bar{b}$ vektorlarga qurilgan uchburchak yuzini hisoblang.

№ **14.** A(2;-1;2), B(1;2;-1), C(3;2;1) nuqtalar berilgan. $\overline{AB} \times \overline{BC}$ ni hisoblang.

№ **15.** Uchlari A (7,3,4), B(1,0,6), C(4,5,-2) nuqtalardan iborat uchburchak yuzini hisoblang.

№ **16.** A (1,2,-1), B(0,1,5), C(-1,2,1), D(2,1,3) nuqtalar bir tekislikda yotadimi?

№ **17.** Quyidagi vektorlar komplanarmi:

a) $\bar{a} = \{-1.3.2\},\ \bar{b} = \{2.-3-4\},\ \bar{c} = \{-3.12.6\}.$

b) $\bar{a} = \{3.-2.1\},\ \bar{b} = \{2.1.2\},\ \bar{c} = \{3.-1.-2\}$

№ **18.** $\bar{a} = \{3.4.0\},\ \bar{b} = \{0.-4.1\},\ \bar{c} = \{0.2.5\}$ vektorlar qanday uchlik tashkil etadi?

№ **19.** Piramidaning uchlari berilgan:

A (2;3;1), B (4;1;-2), C (6;3;7), D (-5;-4;8). Piramidaning D uchidan tushirilgan uzunligini toping.

№ **20.** Ushbu

1) $i \times (j + r) - j \times (i + r) + r \times (i + j + r)$

2) $(a + b + c) \times c + (a + b + c) \times b + (b - c) \times a$

3) $(2a + b) \times (c - a) + (b + c) \times (a + b)$

4) $2i \cdot (j \times r) + 3j \cdot (i \times r) + 4r \cdot (i \times j)$

ifodalar qavslarni ochib soddalashtirilsin.

№ 2**1.** $\bar{a}$ va $\bar{b}$ vektorlar o'zaro 45^0 burchak tashkil etadi. Agar $|\bar{a}| = |\bar{b}| = 5$ bo'lsa, $a - 2b$ va $3a + 2b$ vektorlarda yasalgan uchburchakning yuzi topilsin.

№ **22.** m va n o'zaro 45^0 burchak tashkil etuvchi birlik vektorlar. Diganallari 2m-n va 4m-5n vektorlardan iborat bo'lgan parallelogramm yuzi topilsin.

№ **23.** A (4;2;-3) nuqtaga qo'yilgan $F = \{2.-4.5\}$ kuchning B (3;2;-1) nuqtaga nisbatan kuch momentini toping.

№ **24.** Uchlari A (2,-1,1), B (5,5,4), C (3,2,-1), D (4,1,3,) nuqtalarda bo'lgan piramida hajmini hisoblang.

№ **25.** $\bar{a}=-\bar{i}+3\bar{j}+2\bar{k},\quad \bar{b}=2\bar{i}-3\bar{j}-4\bar{k},\quad \bar{c}=-3\bar{i}+12\bar{j}+6\bar{k}$ vektorlarning o'zaro komplanar ekani ko'rsatilsin. s vektor a va b vektorlar bo'yicha tarqatilsin.

№ **26.** $a=i+j+4r,\quad b=i-2j$ va $c=3i-3j+4r$ vektorlar yasalsin va ular o'zaro komplanar ekanligi ko'rsatilsin. Bu vektorlar orasidagi chiziqli bog'lanish topilsin.

№ **27.** $ABCD$ piramidaning uchlari berilgan:

a) Piramidaning berilgan qirrali orasidagi burchak kosinusini toping;

b) Piramidaning berilgan yoqi yuzini toping;

1) A (6,-4,1), B (6,3,-1), C (2,5,7), D (-4,-3,3) a) AB va AC, b) DBC

2) A (6,4,-7), B (5,7,-4), C (-5,-4,2), D (4,2,3) a) BC va BD ; b) ACD.

3) A (-2,8,7), B (6,-2,-3), C (8,2,-3), D (3,5,3) a) CA va CD; b) BAD.

№ **28.** $\bar{a}=2\bar{i}-3\bar{j}$ va $\bar{b}=3\bar{i}+4\bar{k}$ vektorlarga qurilgan parallelogrammning yuzini toping.

№ **29.** Uchlari A (1,2,0), B (-1,2,1), C (0,-3,2) va D (1,0,1) nuqtalarda bo'lgan piramidani hajmini hisoblang.

№ **30.** $|\bar{a}|=10\quad |\bar{b}|=2\quad \bar{a}\cdot\bar{b}=12\quad [\bar{a}\times\bar{b}]$ $\bar{a}$ va $\bar{b}$ vektorlarni ko'paytmasini uzunligini toping.

№ **31.** $\bar{a}\perp\bar{b}\quad |\bar{a}|=3\quad |\bar{b}|=4$. 1) $|[(\bar{a}+\bar{b})\cdot(\bar{a}-\bar{b})]$, 2) $|[(3\bar{a}-\bar{b})\cdot(\bar{a}-2\bar{b})]$.

№ **32.**

$|\bar{a}|=1\quad |\bar{b}|=2\quad (\bar{a}\bar{b})=\frac{2\pi}{3}.$ 1) $[\bar{a}\bar{b}]^2$, 2) $[(2\bar{a}+\bar{b})\cdot(\bar{a}+2\bar{b})]^2$, 3) $[(\bar{a}+3\bar{b})\cdot(3\bar{a}-\bar{b})]^2$.

№ **33.** $\bar{a}=\{3;-1;-2\}\quad \bar{b}=\{1;2;-1\}.$ 1) $[ab]$, 2) $[(2\bar{a}+\bar{b})\cdot\bar{b}]$,

3) $[2\bar{a}\cdot\bar{b}]$, 4) $[(2\bar{a}-\bar{b})(2\bar{a}+\bar{b})]$

№ **34.** Uchlari A(2;-1;1) , B(5;5;4) , C(3;2;-1) , D(4;1;3) nuqtalarda bo'lgan piramida hajmini toping.

№ **35.** Quyidagi 1) $\bar{a}=3\bar{i}$, $\bar{b}=2\bar{k}$ 2) $\bar{a}=i+j$, $\bar{b}=\bar{i}-\bar{j}$

3) $\bar{a}=2\bar{i}+\overline{3j}$, $\bar{b}=3\bar{j}+2\bar{k}$. hollarda vektorlarga qurilgan parallelogrammning yuzini toping. S = $|a\times b|$

№ **36.** $\bar{a}=2\bar{j}+\bar{k}$ va $\bar{b}=\bar{i}+2\bar{k}$ vektorlarga qurilgan parallelogramning yuzini toping.

S=$|\bar{a}\times\bar{b}|$

№ **37.** Qavslarni ochib, qisqartiring: 1) $\bar{i}\times(\bar{j}+\bar{k})-\bar{j}\times(\bar{i}+\bar{k})+\bar{k}\times(\bar{i}+\bar{j}+\bar{k})$;

2) $(\bar{a}+\bar{b}+\bar{c})\times\bar{c}+(\bar{a}+\bar{b}+\bar{c})\times(\bar{b}-\bar{c})\times\bar{a}$; 3) $(2\bar{a}+\bar{b})\times(\bar{c}-\bar{a})+(\bar{b}+\bar{c})\times(\bar{a}+\bar{b})$

4) $2\bar{i}\times(\bar{j}\times\bar{k})+3\bar{j}\times(\bar{i}\times\bar{k})+4\bar{k}\times(\bar{i}\times\bar{j})$

№ **38.** $\bar{a}\wedge\bar{b}=45^\circ$ $\bar{a}-2\bar{b}$ va $3\bar{a}+2\bar{b}$ vektorlarga qurilgan. S toping, agar $|\bar{a}|=|\bar{b}|=5$

№ **39**. $\bar{a}=3\bar{i}+4\bar{j}$, $\bar{b}=-3\bar{j}+\bar{k}$, $\bar{c}=2\bar{j}+5\bar{k}$ vektorlarga qurilgan parallelogramning hajmini toping .

№ **40.** $\bar{A}(5;\ 2;0)$ $\bar{B}(2;\ 5;\ 0)$ $\bar{C}(1;\ 2;\ 4)$ vektorlar bo'yicha qurilgan piramidaning hajmini toping.

№ **41.** 1) $(\bar{a}+\bar{b})\cdot[(\bar{a}+\bar{c})\times\bar{b}]=-\bar{a}\bar{b}\bar{c}$ 2) $(\bar{a}+2\bar{b}-\bar{c})\cdot[(\bar{a}-\bar{b})\times(\bar{a}-\bar{b}-\bar{c})]=3\bar{a}\bar{b}\bar{c}$ isbotlang.

№ **42.** $\bar{a}=\bar{i}+\bar{j}+4\bar{k}$ $\bar{b}=\bar{i}-2\bar{j}+4\bar{k}$ $\bar{c}=3\bar{i}-3\bar{j}+4\bar{k}$ vektorlarning komplanar ekanligini tekshiring.

№ **43.** O'ng uchlik $\bar{a}\perp\bar{b}\perp\bar{c}$ o'zaro $|\bar{a}|=4$ $|\bar{b}|=2$ $|\bar{c}|=3$ $\bar{a}\times\bar{b}\times\bar{c}=24$

№ **44.** $\bar{c}\perp\bar{a}$ va $\bar{b}$ $(\bar{a}\wedge\bar{b})=30^\circ$ $|\bar{a}|=6$ $|\bar{b}|=3$ $|\bar{c}|=3$

VIII bob.

IKKI VEKTORNING VEKTOR KO'PAYTMASI. UCH VEKTORNING ARALASH KO'PAYTMASI

1 - §. Vektorlarning vektor ko`paytmasining ta'rifi va xossalari

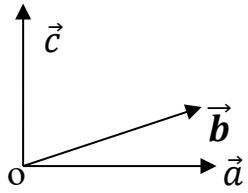
1.1. Ikki vektorning vektoriy ko`paytmasi

Fazoda Dekart koordinata tizimi berilgan bo`lsin. **a** vektorning **b** vektor bilan vektoriy ko`paytmasi **[ab]** yoki **a**x**b** shaklda belgilanadi va quyidagicha aniqlanadi:

1) [**ab**] vektor moduli |**a**|·|**b**| sinφ ga teng bo'lib, bunda φ burchak **a** va **b** lar orasidagi burchak.

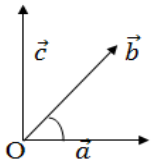
2) [**ab**] vektor **a** va **b** vektorlarning har biriga perprndikulyar.

3) [**ab**] vektor **a** va **b** vektorlarga nisbati shunday yo'nalganki, agar [**ab**] vektorning biror nuqtasidan **a** va **b** vektorlarga qarasak, **a** vektorning **b** vektorga eng qisqa burilish burchagi soat strelkasi harakati yo`nalishiga teskari bo`lib ko`rinsin. Ta'rifdan $\vec{a}$ x $\vec{b}$ $= [\vec{a} \cdot \vec{b}\,] = \vec{c}$ ekanligi kelib chiqadi.



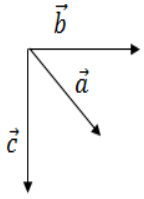
1-chizma

Oxirgi tenglikdan ikki vektorning vektor ko`paytmasi vektor miqdor ekanligi kelib chiqadi. Vektor ko`paytma ta`rifidan quyidagini hosil qilish mumkin $|\vec{c}| = |\vec{a}||\vec{b}|\sin\varphi$.



O`ng uchlik

2-chizma



Chap uchlik

3-chizma

***Ta`rif***. Vektorlar uchligi o`ng (chap) uchlik tashkil qiladi deyiladi, agar uchinchi vektorning uchidan (oxiridan) qaraganda birinchi vektordan ikkinchi

vektorga qisqa burilish soat strekasi yo`nalishiga qarama-qarshi (bo`yicha) bo`lsa (*) tenglik vektor ko`paytmani geometrik ma`nosini anglatadi.

1.2. Vektoriy ko`paytma xossalari.

Geometrik xossalari:

1) agar **a**||**b** bo'lsa [**ab**]=0 va aksincha;

2) agar **a** va **b** lar umumiy boshga keltirilgan bo'lsa, u vaqtda |[**ab**]| son shu vektorlarga ko'rilgan paralellogram yuziga teng

Algebraik xossalari:

1- **Xossa**. [**ab]** = - [**ba] yoki** $\vec{a}\ \mathbf{x}\ \vec{b} = -\vec{b}\mathbf{x}\vec{a}$ Vektor ko`paytmada ko`payuvchi vektorlarni o`rinlari almashtirilsa vektor ko`paytma ishorasi teskarisiga almashadi **.**

2- **Xossa.** [**a**(**b+c**)] = [**ab**]+[**ac**];

3- **Xossa.** $(\mathbf{k}\vec{a})\mathbf{x}\vec{b} = \mathbf{k}(\vec{a}\ \mathbf{x}\ \vec{b})$ O`zgarmas sonni vektor ko`paytma belgisidan tashqariga chiqarish mumkin

4- **Xossa.** Vektorning o`zini o`ziga vektor ko`paytmasi 0 ga teng $\vec{a}\cdot\vec{a} = \mathbf{0}$

Isbot: $\vec{a}$ va $\vec{a}$ ga parallelogram qurib bo`lmaydi. Yoki $\vec{a}$ va $\vec{a}$ ga qurilgan parallelogram yuzasi 0 ga teng bo`ladi.

Endi vektor ko`paytmani o'ziga xos xususiyati, yani uni haqiqiy sonlarni ko'paytirishdan farqini qarab chiqaylik.

5-xossa. Vektor ko'payta assotsiativ emas, ya'ni shunday **a**, **b**, **c**,vektorlar topiladiki, ular uchun $(\boldsymbol{a}\times\boldsymbol{b})\times\boldsymbol{c} \neq \boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})$ o'rinli bo'ladi.

Isbot. Faraz qilaylik **a** va **b** – ixtiyoriy kollerniar bo'lmagan vektorlar bo'lib $\vec{c} = \vec{d}$tenglik o'rinli bo'lsin. U holda $\boldsymbol{a}\times\boldsymbol{b} \neq 0$, va bundan tashqari bu vector **a** va **b** vektorlar yotuvchi tekislikka ortogonal.

Shunday qilib, $\boldsymbol{a}\times\boldsymbol{b}$ va **c** - vektorlar kollerniar emas, shuning uchun $(\boldsymbol{a}\times\boldsymbol{b})\times\boldsymbol{c} \neq 0$. Ikkinchi tomondan, $\boldsymbol{b}\times\boldsymbol{c} = \boldsymbol{b}\times\boldsymbol{b} = 0$. Demak $\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c}) = 0$. Yuqorida kiltirilganlardan $(\boldsymbol{a}\times\boldsymbol{b})\times\boldsymbol{c} \neq \boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})$ ekanligi kelib chiqadi.

Izoh: Vektor ko`paytma ta`rifidan uni bevosita topish mumkin emas. Shuning uchun vektor ko`paytmani koordinata shaklida tasvirlaylik:

$\vec{a}\ \mathrm{x}\,\vec{b} = (a_x\vec{\imath} + a_y\vec{\jmath} + a_z\vec{k})(b_x\vec{\imath} + b_y\vec{\jmath} + b_z\vec{k}) = a_x\mathrm{b_x}\vec{\imath}\vec{\imath} + a_x\mathrm{b_y}\vec{\imath}\vec{\jmath} + a_x\mathrm{b_z}\vec{\imath}\vec{k} +$

$a_y\mathrm{b_x}\vec{\jmath}\vec{\imath} + a_y\mathrm{b_y}\vec{\jmath}\vec{\jmath} + a_y\mathrm{b_z}\vec{\jmath}\vec{k} + a_z\mathrm{b_x}\vec{k}\vec{\imath} + a_z\mathrm{b_y}\vec{k}\vec{\jmath} + a_z\mathrm{b_z}\vec{k}\vec{k} \quad =\gg$

$[\overline{ii}] = \overline{0}, [\overline{ij}] = \overline{k}, [\overline{ik}] = -\overline{j}$

$[\overline{ji}] = -\overline{k}, [\overline{jj}] = 0, [\overline{jk}] = \overline{i}$

$[\overline{ki}] = \overline{j}, [\overline{kj}] = -\overline{i}, [\overline{kk}] = \overline{0}$

$=\gg\ a_x\mathrm{b_y}\vec{k} - a_x\mathrm{b_z}\vec{\jmath} - a_y\mathrm{b_x}\vec{k} + a_y\mathrm{b_z}\vec{\imath} + a_z\mathrm{b_x}\vec{\jmath} - a_z\mathrm{b_y}\vec{\imath} = k\left(a_x\mathrm{b_y} - a_y\mathrm{b_x}\right) +$

$j(a_z\mathrm{b_x} - a_x\mathrm{b_z}) + i(a_y\mathrm{b_z} - a_z\mathrm{b_y})$ **(2*)**

Vektor ko`paytmani hisoblashda (2*) formuladan foydalanaish ancha noqulay. Hisoblash uchun qulay formulani keltirib chiqaraylik, har bir qavs ichidagi ayirma qiymati qandaydir songa teng. (2*) formulani ikkinchi tartibli aniqlovchidan foydalanib quyidagicha yozamiz.

$$\vec{a}\ \mathrm{x}\,\vec{b} = \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix}\vec{\imath} - \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix}\vec{\jmath} + \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}\vec{k} \qquad (3^*)$$

(6*) formuladan qulay formula tuzamiz:

$$\vec{a}\ \mathrm{x}\,\vec{b} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \qquad (4^*)$$

Misol : $\vec{a} = \vec{\imath} + \vec{\jmath} - \vec{k};\ \vec{b} = 2\vec{\imath} + \vec{\jmath} + 3\vec{k}$

$$\vec{c} = \vec{a}\ \mathrm{x}\,\vec{b} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 3\vec{\imath} - 2\vec{\jmath} + \vec{k} - 2\vec{k} + \vec{\imath} - 3\vec{\jmath} = 4\vec{\imath} - 5\vec{\jmath} - \vec{k}$$

$\vec{c} = \{4; -5; -1\}$

Masala. $\bar{a}$ va $\bar{b}$ vektorlar o'zaro $\phi = \frac{\pi}{6}$ li burchakni tashkil qilib, $\bar{a} = 6\ \vec{b} = 5$ bo'lsa, $\left|\bar{a} \times \vec{b}\right|$ toping.

Echish. Vektor ko'paytma moduli $\left|\bar{a} \times \vec{b}\right| = |\bar{a}|\left|\vec{b}\right| sin\,\phi$ formula yordamida hisolanishini inobatga olsak $\left|\bar{a} \times \vec{b}\right| = |\bar{a}|\left|\vec{b}\right| sin\,\phi = 30 \cdot \frac{1}{2} = 15$ ga rga bo'lamiz.

Masala. Agar $|\vec{a}| = 1, |\vec{b}| = 2, \phi = (\vec{a}\ \vec{b}) = \frac{2\pi}{3}$ bo'lsa $[\bar{a} \times \vec{b}]^2$, $[(2\bar{a} + \vec{b}) \times (\bar{a} + 2\bar{b})]^2$ va $[(\bar{a} + 3\vec{b}) \times (3\bar{a} - \bar{b})]^2$ larni toping.

Echish.

1) $[\bar{a} \times \vec{b}]^2 = (\bar{a}\ \vec{b}\ sin\phi)^2 = (2 \cdot \frac{\sqrt{3}}{2})^2 = (\sqrt{3})^2 = 3$

2) $[(2\bar{a} + \vec{b}) \times (\bar{a} + 2\bar{b})]^2 = (2\bar{a}^2 + 4\bar{a}\bar{в} - \bar{a}\bar{b} + 2\bar{b}^2)^2 = (3|\bar{a}||\bar{b}|\ sin\phi)^2 = (6\frac{\sqrt{3}}{2})^2 = 9 \cdot 3 = 27$

3) $[(\bar{a} + 3\vec{b}) \times (3\bar{a} - \bar{b})]^2 = (3\bar{a}^2 - \bar{a}\ \vec{b} - 9\bar{a}\bar{b} - 3\bar{b}^2)^2 = (-10 \cdot 2\frac{\sqrt{3}}{2})^2 = 100 \cdot 3 = 300$

1.3. Ikki karrali vektor ko'paytma ta'rifi va xossalari

Ushbu $\vec{a}, \vec{b}, \vec{c}$ uch vektorni quyidagi qo'sh ko'paytmalarini qaraylik $\vec{a} \cdot (\vec{b}\ x\ \vec{c})$, $\vec{a}\ x(\vec{b} \cdot \vec{c})$, $\vec{a} \cdot (\vec{b} \cdot \vec{c})$ va $\vec{a}\ x(\vec{b}\ x\ \vec{c})$. Bu ko'paytmalardan birinchisi skalyar miqdor, ikkinchisi ma'noga ega emas, chunki vektorni skalyarga vektorial ko'paytirish amali uchunchisi $\vec{a}$vektorga kolleniar vector bo'ladi oxirgisi esa hozircha nom'alum bolib, uni o'rganaylik.

Ta'rif $[\overline{a}[\overline{b}\overline{c}]]$ ga, $\overline{a}$vektorni $\overline{b}$va $\overline{c}$ vektorlarni vector ko'paytmasi $[\overline{b}\overline{c}]$ga, ikki karrali (qo'sh) vector ko'paytmasi deyiladi. Ikki karrali vekctor ko'paytma $\overline{a}x(\overline{b}x\overline{c})$ kabi ham belgilanadi.

Ikki karrali vekctor ko'paytma $[\overline{a}[\overline{b}\overline{c}]]$ quyidagicha hisoblanadi, $\vec{a}\ x(\vec{b}\ x\ \vec{c}) = \vec{b} \cdot (\vec{a} \cdot \vec{c}) = \vec{a} \cdot (\vec{b} \cdot \vec{c})$ formula bilan hisoblanadi.

Ikki karrali vector ko'paytma ixtiyoriy $\overline{a}$,$\overline{b}$va $\overline{c}$vektorlar uchun quyidagi xossalarga ega.

1. $[\overline{a}[\overline{b}\overline{c}]] = (\overline{a}, \overline{b}) \cdot \overline{c} - \overline{a} \cdot (\overline{b}, \overline{c})$
2. $[\overline{a}[\overline{b}\overline{c}]] = [\overline{b}[\overline{c}\overline{a}]] + [\overline{c}[\overline{b}\overline{a}]] = \overline{a}$
3. $\vec{b} = \frac{(\vec{a},\vec{b})}{|\vec{a}|^2} \cdot \vec{a} + \frac{1}{|\vec{a}|^2}[\vec{a}, [\vec{b}, \vec{a}]], |\vec{a}| \neq 0$

4. $\vec{a} = \{x_a, y_a, z_a\}^T, \vec{b} = \{x_b, y_b, z_b\}^T, \vec{c} = \{x_c, y_c, z_c\}^T$ lar, $\vec{a}, \vec{b}, \vec{c}$ vektorlarning standart bazisdagi koordinatalari bo'lsa, bu vektorlarning ikki karrali vector ko'paytmasi $\vec{d} = \left[\overline{a}\left[\overline{b}\overline{c}\right]\right]$ning $\vec{d} = \{x_d, y_d, z_d\}$ biror bazisdagi koordinatalari $\vec{d} = b \cdot c^T \cdot a = c \cdot a^T \cdot b$ mnemonic qoida "bats"- "tsab" ga asosan topiladi.

Izoh. 1- xossaning isbotlashda skalyar va vector ko'paytmalarni hisoblash qoidalaridan foydalaniladi.

2 - xossaning isboti, 1- xossadan $\left[\overline{b}[\overline{c}\overline{a}]\right] = (\overline{b}, \overline{c}) \cdot \overline{a} - (\overline{b}, \overline{c}) \cdot \overline{a}$ va$\left[\overline{c}\left[\overline{b}\overline{a}\right]\right] = (\overline{b}, \overline{c}) \cdot \overline{a} - (\overline{c}, \overline{a}) \cdot \overline{b}$ larni davriy o'rin lmashtirish orqali, skalyar ko'paytma kommutativlik xossasini inobatga olib ularni ayirish orqali ko'rsatiladi.

3-xossaning o'rinli ekanligi, agar $\overline{c} = \overline{a}$ deb olsak, 1-xossadan kelib chiqadi. Bu tenglik ixtiyoriy $\overline{b}$ vektorni ortonormal proyeksiyasi $pr_{\overline{a}}\overline{b} = \frac{(\overline{a},\overline{b})}{|\overline{a}|^2} \cdot \overline{a}$ va ortonormal tashkil etuvchisi $\overline{b}_{-\overline{a}} = \frac{1}{|\overline{a}|^2} \cdot \left[\overline{c}\left[\overline{b}\overline{a}\right]\right]$ lari yig'ndisi shaklida, $\overline{a}$ vector o'qi bo'yicha yoyish imkonini beradi.

1.4. Ikki karrali vektor ko'paytmani koordinata shakli

Faraz qilaylik $\vec{a}$, $\vec{b}$, $\vec{c}$vektorlar koordinatalari bilan berilgan bo'lsin ya'ni $\vec{a} = \{a_x, a_y, a_z\}$, $\vec{b} = \{b_x, b_y, b_z\}$ va $\vec{c} = \{c_x, c_y, c_z\}$ u holda ularning qo'sh vector ko'paytmasi

$$\left[\vec{a}\left[\vec{b}\ \vec{c}\right]\right] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_y c_z - b_z c_y & -b_x c_z + b_z c_x & b_x c_y - b_y c_x \end{vmatrix} \quad (')$$

ko'rinishda bo'ladi.

Misol. $\vec{a} = \{2,3,1\}$, $\vec{b} = \{0,-2,5\}$ va $\vec{c} = \{4,3,2\}$ vektorlani qo'sh vector ko'paytmasini hisoblaylik.

Echish. (') dan foydalanamiz

$$\left[\vec{a}\left[\vec{b}\ \vec{c}\right]\right] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_y c_z - b_z c_y & -b_x c_z + b_z c_x & b_x c_y - b_y c_x \end{vmatrix} =$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 1 \\ -2\cdot 2-5\cdot 3 & -0\cdot 2+5\cdot 40 & 3-(-2)\cdot 4 \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 1 \\ -19 & 20 & 8 \end{vmatrix} =$$

$= 24\vec{i}$ -19$\vec{j}$ + 40$\vec{k}$ +57$\vec{k}$ -20$\vec{i}$ -16$\vec{j}$ = $4\vec{i}$ - 35$\vec{j}$ + 97$\vec{k}$ = $\{4, -35, 97\}$

2 - §. Vektor algebrasining fizikaga ba'zi tatbiqlari

1. A nuqtaga qo'yilgan $\vec{F}$ kuchning biror bir O nqtaga nisbatan kuch momenti, $\overrightarrow{OA}$ va $\vec{F}$ vektorlarning vector ko'paytmasiga teng, ya'ni

$$\vec{M} = \left[\overrightarrow{OA}, \vec{F}\right]$$

2. Qo'zgarmas o'q atrofida burchak tezligi bilan harakatlanayotgan nuqtaning chiziqli tezligi, burchak tezligi vektori va qo'zfalmas nuqta radius vektorinig vector ko'paytmasiga teng, ya'ni, $\vec{v} = \left[\vec{w}, \vec{R}\right]$

3. Aylanma harakatda $\vec{w}$ burchak tezligi bilan harakatlanuvchi jismga va $\vec{v_e}$, tezlikka ega nisbiy harakatga ta'sir etuvchi Koriolis kuchi, shu vektorlarning vector ko'paytmasiga to'g'ri proptsionaldir $\overrightarrow{F_k} = 2m[\vec{v_e}, \vec{w}]$ **,** bu erda **m** – jism massasi.

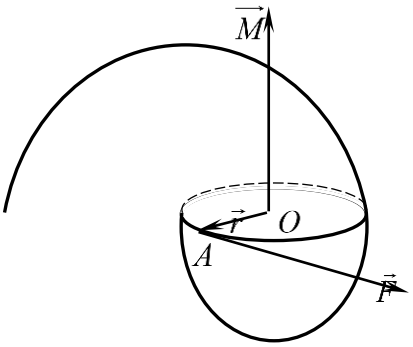
Masalan, shimolga qarab $\varphi = 45$ gradiusli meridianda 100 km.soat tezlik bilan harakatlanayotgan, og'rligi 1000 кг bo'lgan avtomibilga ta'sir etuvchi Koriolis kuchi, janubga yo'nalgan va kattaligi

$$\left|\overrightarrow{F_k}\right| = 2m * |\vec{v_e}| * |\vec{w}| * \sin\varphi \text{ ga teng.}$$

4. Magnit maydonida $\vec{v}$ tezlik bilan harakatlanayotgan**,** q elektr zaryadli zarrachaga ta'sir qiluvchi Lorens kuchi, tezlik vektori va magnit maydoni kuch xarakteristikasi bo'lgan, $\vec{B}$ magnit induksiyasi vektori, vector ko'paytmasiga proportsional bo'ladi, ya'ni $\overrightarrow{F_л} = q\left[\vec{v}, \vec{B}\right]$

Nuqtaga nisbatan kuch momenti

O nuqtasi mahkamlangan qattiq jism A nuqtasiga qo‘yilgan $\vec{F}$ kuch ta’sirida Onuqta atrofida aylanma harakat qilayotgan bo‘lsin (1-chizma).



1-chizma

Fizika kursidan ma’lumki $\vec{F}$ kuchning O nuqtaga nisbatan momenti deb O nuqtadan o‘tuvchi va quyidagi shartlarni qanoatlantiruvchi $\vec{M}$vektorga aytiladi:

1) $\vec{M} \perp \vec{r}$ va $\vec{M} \perp \vec{F}$, bu yerda $\vec{r} = \overrightarrow{OA} - A$ nuqtaning radius vektori;

2) $|\vec{M}| = |\vec{r}| \cdot |\vec{F}| \sin\phi$, bu yerda $\phi = (\vec{r}\hat{,}\vec{F})$;

3) $\vec{r}, \vec{F}, \vec{M}$ vektorlar o‘ng uchlik tashkil qiladi.

Shunday qilib, $\vec{M} = \vec{r} \times \vec{F}$, ya’ni qo‘zg‘almas nuqtaga nisbatan kuch momenti kuch qo‘yilgan nuqta radius vektorining kuch vektoriga vektor ko‘paytmasiga teng. Bu jumla ***vektor ko‘paytmaning mexanik ma’nosini*** anglatadi.

Masala. O nuqtasi mahkamlangan qattiq jism $A(1,2,3)$ nuqtasiga qo‘yilgan $\vec{F} = 3\vec{i} + 2\vec{j} - \vec{k}$ kuch ta’sirida Onuqta atrofida aylanma harakat qilaqilayotgan bo‘lsa, shu kuchning kuch monentini toping.

Echish. $\vec{M} = \vec{r} \times \vec{F}$, $\vec{r} = \{1,2,3\}$ va $\vec{F} = \{3,2,-1\}$ bo’lgani uchun,

$$\vec{M} = [\vec{r}\ \vec{F}] = \left\{\begin{vmatrix} Y_1 & Z_1 \\ Y_2 & Z_2 \end{vmatrix}, \begin{vmatrix} X_1 & Z_1 \\ X_2 & Z_2 \end{vmatrix}, \begin{vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{vmatrix}\right\} =$$

$\left\{\begin{vmatrix} 2 & 3 \\ 2 & -1 \end{vmatrix}, \begin{vmatrix} 1 & 3 \\ 3 & -1 \end{vmatrix}, \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}\right\}$=$\{-8,-10,-4\}$, Uning kattaligi esa

$$\left|\vec{M}\right| = \left|[\vec{r}\ \vec{F}]\right| = |\{-8,-10,-4\}| = \sqrt{(-8)^2 + (-10)^2 + (-4)^2} = \sqrt{64 + 100 + 16} = \sqrt{180}$$

2.2. Aylanma harakat chiziqli tezligini topish

Yuqorida keltirilgandagi kabi qo‘zg‘almas O nuqta atrofida $\vec{\omega}$ burchak tezlik bilan aylnma harakat qilayotgan qattiq jism Mnuqtasining chziqli tezligi Eyler

formulasi bilan topiladi: $\vec{v} = \vec{\omega} \times \vec{r}$, bu yerda $\vec{r} = \overrightarrow{OM} - M$nuqtaning radius vektori.

Masala 1.

m, n ning qanday qiymatlarida $\vec{a} = \{-2; 3; n\}$ va $\vec{b} = \{m; -6; 2\}$ vektorlar kollinear bo‘ladi?

Echish.

Ikki vektorning kollinearlik sharti $\frac{a_x}{b_x} = \frac{a_y}{b_y} = \frac{a_z}{b_z}$ ga ko‘ra $\frac{-2}{m} = \frac{3}{-6} = \frac{n}{2}$ ni topamiz. Bundan, esa nma’lumlarni qiymatlarini aniqlaymiz, ya’ni $m = 4, n = -1$.

Masala 2. $\vec{a} = 2\vec{j} - 3\vec{k}$ va $\vec{b} = 4\vec{i} + 3\vec{j}$ vektorlarga qurilgan parallelegramning yuzini hisoblaymiz

Echish. Parallelegramning yuzini hisoblash formulasini koordinata ko’rinishidan foydalanamiz. Parallelegram tomonlarini tashkil etuvchi vektorlar o’zlarinig koordinatalari bilan berilgan bo’lsa, ya’ni $\vec{a} = \overrightarrow{a_x}\,\vec{i} + \overrightarrow{a_y}\,\vec{j} + \overrightarrow{a_z}\,k$ va $\vec{b} = \overrightarrow{b_x}\,\vec{i} + \overrightarrow{b_y}\,\vec{j} + \overrightarrow{b_z}\,k$bo’lsa $mod(\vec{a} \times \vec{b}) = mod \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$ yoki

$$|[ab]| = \left\{ \begin{vmatrix} Y_1 & Z_1 \\ Y_2 & Z_2 \end{vmatrix}, \begin{vmatrix} X_1 & Z_1 \\ X_2 & Z_2 \end{vmatrix}, \begin{vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{vmatrix} \right\} = \sqrt{\begin{vmatrix} Y_1 & Z_1 \\ Y_2 & Z_2 \end{vmatrix}^2 + \begin{vmatrix} X_1 & Z_1 \\ X_2 & Z_2 \end{vmatrix}^2 + \begin{vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{vmatrix}^2}$$ (A) bo’ladi.

(A) ga asosan

$$S = \sqrt{\begin{vmatrix} 2 & -3 \\ 3 & 0 \end{vmatrix}^2 + \begin{vmatrix} 0 & -3 \\ 4 & 0 \end{vmatrix}^2 + \begin{vmatrix} 0 & 2 \\ 4 & 3 \end{vmatrix}^2} = \sqrt{9^2 + 12^2 + (-8)^2} = 17(y.b.)$$. ni topamiz.

3- §. UCH VEKTORNING ARALASH KO`PAYTMASI

3.1. Uch vektorning aralash ko`paytmasi.

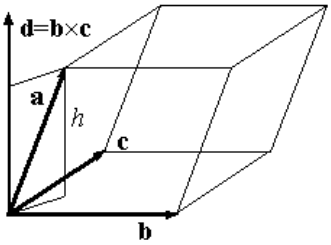
a, b, c vektorlar berilgan bo`lsin, **a** vektor **b** vektor bilan vektoriy ko`paytirilsin, hosil bo`lgan vektor esa **c** vektor bilan skalyar ko`paysin. Bunday ko`paytma vektor - skalyar yoki aralash ko`paytma deyiladi va **[ab]c** kabi belgilanadi.

Ta'rif **a,b,c** vektorlarning aralash ko'paytmasi deb $\left(\vec{a}x\vec{b}\right)\cdot\vec{c}$ yoki $\boldsymbol{a}\cdot(\boldsymbol{b}\times\boldsymbol{c})$ songa aytiladi. Aralash ko'paytma **abc** kabi belgilanadi.

Ta'rifdan $\left(\vec{a}x\vec{b}\right)\cdot\vec{c}$ skalyar miqdor ekanligi kelib chiqadi. $\left(\vec{a}x\vec{b}\right)\cdot\vec{c}$ ning qiymati ko'payuvchi vektorlarga qurilgan parallelepipedning hajmiga teng.

Ammo yozuvda qisqa qilib **abc** deb yoziladi. Agar berilgan vektorlarning qaysi birinchi qaysi ikkinchi va qaysi uchinchi ekani aytilgan bo`lsa, u vaqtda tartiblangan uchlik vektor berilgan deyiladi.

Geometrik nuqtaiy nazardan, aralash ko'paytma absolyut qiymati, $\vec{a},\vec{b},\vec{c}$ vektorlarga qurilgan parallelepipedning hajmiga teng, ya'ni



1 – chizma

$V=\left|\left(\vec{a}x\vec{b}\right)\cdot\vec{c}\right|$ ((1) - chizma)

Masala. Agar $|\vec{a}|=3,\left|\vec{b}\right|=6,|\vec{c}|=2,\quad\alpha=(\vec{c},\vec{a})=135°,\beta=(\vec{b},[\vec{c},\vec{a}])=45°$ bo'lsa $\vec{a},\vec{b},\vec{c}$ vektorlarga qurilgan parallelepipedning hajmini hisoblang.

Echish. $V=\left|\left(\vec{c}\;x\;\vec{a}\right)\cdot\vec{b}\right|$ formuladan foydalanib $V=\left|(\vec{c}\;x\;\vec{a})\cdot\vec{b}\right|=\left|\vec{c}\;x\;\vec{a}\right|\cdot\left|\vec{b}\right|\cdot cos\,\beta$ ==$\left|\vec{c}\right|\cdot\left|\vec{a}\right|\cdot sin\,\alpha\cdot\left|\vec{b}\right|\cdot cos\,\beta$=$2\cdot3\cdot6\cdot sin\,135°\cdot cos\,45°$=$2\cdot3\cdot6\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}=$

$=18$ (kub birlik)

Teorema Uch vector aralash ko'paytmasi nolga teng, faqat va faqat qachonki agar **a**,**b**,c vektorlar komplanar bo'lsa.

Isbot. Ta'rifa ko'ra $\boldsymbol{abc}=\boldsymbol{a}\cdot(\boldsymbol{b}\times\boldsymbol{c})$ bo'ladi. Skalyar ko'paytmaning 8-xossasiga asosan $\left(\vec{a}x\vec{b}\right)\cdot\vec{c}$=0 bo'ladi, faqat va faqat, agar **a** va $\boldsymbol{b}\times\boldsymbol{c}$ vektorlar perpendikulyar bo'lsa.

Agar $\boldsymbol{b}\times\boldsymbol{c}\neq0$ bo'lsa, u holda $\boldsymbol{b}\times\boldsymbol{c}$ vector **b**,**c** vektorlar yotuvchi tekislikka perpendikulyar bo'ladi, demak **a** vector **b**,**c** vektorlar joylashgan tekislida yotadi.

Agar $\boldsymbol{b}\times\boldsymbol{c}=0$ bo'lsa, u holda **b** va **c** vektorlar kolleniar, u holda **a**,**b**,**c** vektorlar komplanar bo'ladi.

Masalan**:** **a,b,c** deb yozilgan bo`lsa, demak **a** birichi, b-ikkinchi va **c**-uchinchidir. Agar uchta **a,b,c** nokomplanar vektor umumiy boshga keltirilganda

o`ng qo`lning katta, kursatkich va o`rta barmoqlari kabi joylashsa, bunday vektorlar o`ng vektorlar deyiladi. Shuningdek chap vektorlar tushunchasi ham kiritiladi.

3.2. Aralash ko`paytmaning xossalari.

1) Aralash ko'paytmaning istalgan ikkita ko'paytuvchisining o'rinlari o'zaro almashtirilsa, ko'paytmaning ishorasi o'zgaradi:

$$\left(\bar{a}\times\bar{b}\right)\cdot\bar{c}=-\left(\bar{a}\times\bar{c}\right)\cdot\bar{b}=-\left(\bar{c}\times\bar{b}\right)\cdot\bar{a}.$$

2) Agar berilgan uchta vektordan ikkitasi o'zaro teng yoki parallel bo'lsa, aralash ko'paytma 0 ga teng bo'ladi.

3) «Nuqta bilan ko'rsatilgan va «krest» (x) bilan ko'rsatilgan amallarning o'rinlarini almashtirish mumkin:

$\left(\vec{a}x\vec{b}\right)\cdot\vec{c}=\vec{a}\cdot(\vec{b}\ x\ \vec{c})$ shuning uchun ham aralash ko'paytmalarni $\vec{a}\cdot\vec{b}\cdot\vec{c}$ko'rinishda, ya'ni qavslarni va amallar belgilarini ko'rsatmasdan yozish qabul qilingan.

4) $\vec{a}\cdot\vec{b}$ va $\vec{c}$ vektorlarda yasalgan parallelopipedning hajmi:

$$V=\pm\,\vec{a}\ \vec{b}\ \vec{c}=\begin{cases}+vektorlar\ \rightleftarrows\ o'nguchliktashkiletsa\\-vektorlar\ \rightleftarrows\ chapuchliktashkiletsa\end{cases}$$

$\vec{a}\cdot\vec{b}$ va $\vec{c}$ vektorlarda yasalgan piramidaning hajmi: $V=\pm\frac{1}{6}\left(\left|\vec{a}\cdot\vec{b}\cdot\vec{c}\right|\right)$ kabi hisoblanadi.

5) Komplanarlik sharti.

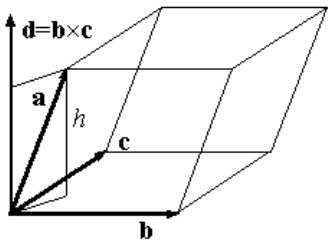
Agar $\vec{a}\cdot\vec{b}$va $\vec{c}$ vektorlar o'zaro komplanar bo'lsa, $\vec{a}\ \vec{b}\ \vec{c}=0$ va aksincha, so'nggi tenglik bajarilsa, berilgan uch vektor o'zaro komplanar bo'ladi. Shuning bilan birga $\vec{a}\cdot\vec{b}$va $\vec{c}$orasida $\vec{c}=m\vec{a}+n\vec{b}$ ko'rinishdagi chiziqli bog'lanish mavjud bo'ladi.

4- §. Koordinata shakidagi aralash ko`paytma.

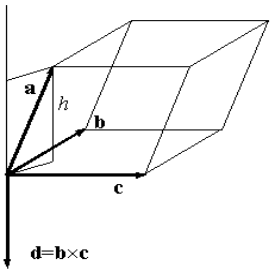
4.1. Uch vektorning komplanarlik sharti

Agar uch vektor komplanar vektorlar bo`lsa, ular o`ng uchlik ham, chap uchlik ham hisoblanmaydi.

Agar **a,b,c** - o'ng uchlik tashkil qilsa, u vaqtda **b,c,a** va **c,a,b** ham o`ng uchlik bo`ladi. Bu holda **b,a,c ; a,c,b** va **c,b,a** lar chap uchlik tashkil qiladi.



O'ng uchlik 1- chizma



Chap uchlik 2- chizma

Teorema. Agar **a,b,c** o`ng uchlik bo`lsa,u vaqtda **abc** aralash ko`paytma qiymati shu vektorlarga qo`rilgan parallelopiped hajmiga teng bo`ladi; agarda bu vektorlar chap uchlik tashkil qilsa, u vaqtda **abc** minus ishora bilan olingan shu parallelopiped hajmiga teng bo`ladi. Agarda **a,b,c** lar komplanar bo`lsa **abc**=0 bo`ladi.

Izoh: Aralash ko`paytmaning son qiypmti berilgan, uchta vektorga qurilgan parallelopipedning hajmiga teng bo`ladi.

**Ta`rif:** Fazoda berilgan uchta 0 bo`lmagan vektorlar komplanar vektorlar deyiladi. Agar ular 1ta yokiparallel tekislikda yotsa . Uchta komplanar vektorga parallelepiped qurib bo'lmaydi, demak komplanar vektorlarga qurilgan parallepipedning hajmi nol'ga teng ekan.

4.2. Koordinata shaklidagi aralash ko'paytma

a = {X_1, Y_1, Z_1}, **b** = {X_2, Y_2, Z_2} ва **c** = {X_3, Y_3, Z_3} vektorlar berilgan bo'lsin. U holda **abc** aralash ko'paytma $\overline{abc} = \begin{vmatrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ X_3 & Y_3 & Z_3 \end{vmatrix}$ formula bilan aniqlanadi.

Isbot. $[ab] = \left\{ \begin{vmatrix} Y_1 & Z_1 \\ Y_2 & Z_2 \end{vmatrix}, \begin{vmatrix} X_1 & Z_1 \\ X_2 & Z_2 \end{vmatrix}, \begin{vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{vmatrix} \right\}$ (1) va **c**={X_3, Y_3, Z_3} lardan skalyar ko'paytma ta'rifiga asosan

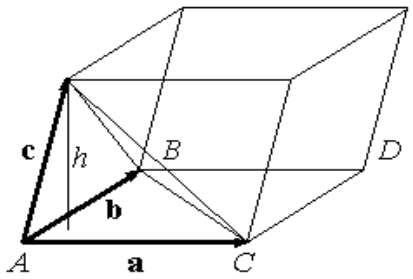
$$[\overline{a}\overline{b}] \cdot \overline{c} = \overline{a}\overline{b}\overline{c} = \begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix} \cdot X_3 - \begin{vmatrix} x_1 & z_1 \\ x_2 & z_2 \end{vmatrix} \cdot Y_3 + \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} \cdot Z_3 = \begin{vmatrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ X_3 & Y_3 & Z_3 \end{vmatrix}$$

ni topamiz, ushbudan teorema isboti kelib chiqadi.

Uchta nol bo’lmagan **a,b,c** vektorlarning komplanarlik sharti $\begin{vmatrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ X_3 & Y_3 & Z_3 \end{vmatrix} = 0$

tenglikdan iboratdir (bu shartning bajarilishi zarur va etarlidir).

Izoh. **a**={X_1, Y_1, Z_1}, **b**={X_2, Y_2, Z_2} ва **c**={X_3, Y_3, Z_3}vektorlarga qurilga parallelepiped hajmi $V_p = |\overline{abc}| = \left|\begin{matrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ X_3 & Y_3 & Z_3 \end{matrix}\right|$ formula bilan hisoblanadi, tetraedr – uchburchakli piramida hajmi esa $V_t = \frac{1}{6}|\overline{abc}| = \frac{1}{6}\left|\begin{matrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ X_3 & Y_3 & Z_3 \end{matrix}\right|$kabi aniqlanadi (3-chizma).



3-chizma

XULOSA

Ushbu qo’llanmani yozishda asosiy e’tibor, vektorlarga bag’ishlangan mavzularni qa’tiy ketma-ketlikda bayon qilishga qaratildi. Ikkinchidan qaralgan tushunchalarni matematika fanini talabalar tomonidan chuqur o’rganish imkonini hosil qilish ko’nikmalariga e’tibor berildi.

Vektorlar nazariyasi apparatidan foydalanib fizika kursida uchraydigan ba’zi masalalar echib ko’satilgah va bu kursdagi ba’zi masalalarni echish uchun formulalar keltirilgan.

Ushbu qo’llanmada innovatsion ta’lim texnologiyalari hamda ilg`or tajribalarga ilmiy yondoshib, mavzuga tegishli ma’lumotlar soda va talabalarga tushunarli bayon qilingan.

Shu bilan bir qatorda mavzu “Matematika” fanini xususiyatidan kelib chiqqan holda, innovatsion ta`lim texnologiyalari va axborot-kommunikatsiya texnologiyalari asosida matematik qa’tiyatlik bilan bayon etilgan.

Qo’llanmada vektoriy va geometrik hamda algebraik talqini keltirilgan, hu mavzularga tegishli teoremalar isbotlari berilgan.

Ikki vektorning skalyar, vektor va uch vektorning aralash ko’paytmalarining ta’riflari, xossalari, geometric va fizik ma’nolari hamda ularning koordinata shakllari batafsil bayon qilingan.

Vektorning skalyar, vektor va aralash ko’paytmalari yordamida turli masalalar yechib ko’rsatilgan. Vektor algebrasiga oid fomulalarni keltirilishi, vektorlar nazariyasi tushunchalarini qo’llab masalalar yechishga yordam beradi umud qilamiz.

Vektorlar nazariyasdagi masalalarni, ana’naviy klassik usulda yechishga, nisbatan Maple paketi yordamida yechish, ancha qulayligi, ba’zi masalalarni Maple paketi yordamida AKTni qollab yechish orqali ko’rsatilgan.

Vektor algebrasinig asosiy formulalari

1-jadval

№	Nomlanishi	Vektor ko'rinishida hisoblash	Koordinata ko'rinishida hisoblash
1	Vektor	$\overline{a} = a_x \vec{i} + a_y\overline{j} + a_z\overline{k}$ $\overline{b} = b_x \vec{i} + b_y\overline{j} + \overline{b_z}\overline{k}$ $\overline{c} = c_x \vec{i} + c_y\overline{j} + \overline{c_z}\overline{k}$	$\vec{a} = \{a_x; a_y; a_z\}$ $\overline{b} = \{b_x; b_y; b_z\}$ $\overline{c} = \{c_x; c_y; c_z\}$
2	Vektor moduli	$\lvert\overline{a}\rvert = \sqrt{\overline{a}^2}$	$\lvert\overline{a}\rvert = \sqrt{{a_x}^2 + {a_y}^2 + {a_z}^2}$
3	Vektorning yo'naltiruvchi kosinuslari	$cos\,\alpha = \frac{x}{\lvert\overline{a}\rvert}$, $cos\,\beta = \frac{y}{\lvert\overline{a}\rvert}$, $cos\,\gamma = \frac{z}{\lvert\overline{a}\rvert}$	$cos\,\alpha = \frac{x}{\sqrt{x^2+y^2+z^2}}$, $cos\,\beta = \frac{y}{\sqrt{x^2+y^2+z^2}}$, $cos\,\gamma = \frac{z}{\sqrt{x^2+y^2+z^2}}$
4	Vektorning ortlari	$\overline{a^o} = \frac{\overline{a}}{\lvert\overline{a}\rvert}$	$\overline{a^o} = \{cos\,\alpha, cos\,\beta, cos\,\gamma\}$
5	Vektorni ixtiyoriy bazisda yoyish	$\overline{a} = \alpha \cdot \overline{b} + \beta \cdot \overline{c}$ $\overline{a} = \alpha \cdot \overline{b} + \beta \cdot \overline{c} + \gamma\overline{d}$	$\begin{cases} a_x = \alpha \cdot b_x + \beta \cdot c_x \\ a_y = \alpha \cdot b_y + \beta \cdot c_y \end{cases}$ $\begin{cases} a_x = \alpha \cdot b_x + \beta \cdot c_x + \gamma \cdot d_x \\ a_y = \alpha \cdot b_y + \beta \cdot c_y + \gamma \cdot d_y \\ a_z = \alpha \cdot b_z + \beta \cdot c_z + \gamma \cdot d_z \end{cases}$

2-jadval

№	Nomlanishi	Vektor ko'rinishida hisoblash	Koordinata ko'rinishida hisoblash
1	Vektorlarni qo'shish	$\overline{a} + \overline{b}$	$\{a_x + b_x; a_y + b_y; a_z + b_z\}$
2	Vektorlarni ayirish	$\overline{a} - \overline{b}$	$\{a_x - b_x; a_y - b_y; a_z - b_z\}$

3	Vektorlarni ckalyar (son) ga ko'paytirish	$\lambda\,\vec{a}$	$\lambda\,\vec{a} = \{\lambda a_x; \lambda a_y; \lambda a_z\}$
4	Skalyar ko'paytirish	$\vec{a} \cdot \vec{b} = \lvert\vec{a}\rvert \cdot \lvert\vec{b}\rvert \cdot cos\,\phi$	$\vec{a} \cdot \lvert\vec{b}\rvert = a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z$
5	Vektor ko'paytirish	$\left[\vec{a} \cdot \vec{b}\right] = \vec{c}$ $\lvert\vec{c}\rvert = \lvert\vec{a}\rvert \cdot \lvert\vec{b}\rvert \cdot sin\,\phi$ $\vec{c} \perp \vec{a}, \vec{c} \perp \vec{b}$	$\vec{a}\ \mathrm{x}\ \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$
6	Aralash ko'paytirish	$\vec{a} \cdot \vec{b} \cdot \vec{c} = (\left[\vec{a}, \vec{b}\right], \vec{c})$	$\overline{a}\overline{b}\overline{c} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$

3-jadval

1	Vektorlarning tenglik sharti	$\overline{a} = \overline{b}$	$a_x = b_x; a_y = b_y; a_z = b_z$
2	Vektorlarning kolleniarlik sharti	$\overline{a} = \lambda \cdot \overline{b}$	$\frac{a_x}{b_x} = \frac{a_y}{b_y} = \frac{a_z}{b_z}$
3	Vektorlarning perpendikulyarlik sharti	$(\overline{a}, \overline{b})$	$a_x \cdot b_x + a_y b_y + a_z \cdot b_z$
4	Vektorlarning komplanarlik sharti	$\overline{a}\overline{b}\overline{c} = 0$	$\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = 0$

4-jadval

№	Nomlanishi	Vektor ko'rinishida va koordinata ko'rinishida hisoblash
1	Vektor moduli	$\lvert\overline{a}\rvert = \sqrt{\overline{a}^2}, \quad \lvert\overline{a}\rvert = \sqrt{{a_x}^2 + {a_y}^2 + {a_z}^2}$
2	Vektorlar orasidagi burchak kosinuso	$cos\,\phi = \frac{\vec{a} \cdot \vec{b}}{\lvert\vec{a}\rvert \cdot \lvert\vec{b}\rvert} = \frac{a_x \cdot b_x + a_y b_y + a_z \cdot b_z}{\sqrt{{a_x}^2 + {a_y}^2 + {a_z}^2} \cdot \sqrt{{b_x}^2 + {b_y}^2 + {b_z}^2}}$
3	Vektorning proeksiasi	$Pr_{\vec{b}}\,\vec{a} = \frac{\vec{a} \cdot \vec{b}}{\lvert\vec{b}\rvert} = \frac{a_x \cdot b_x + a_y b_y + a_z \cdot b_z}{\sqrt{{b_x}^2 + {b_y}^2 + {b_z}^2}}$

4	Parallelogramning yuzi	$S_{paral-gram} = \left\|\left[\vec{a} \cdot \vec{b}\right]\right\| = \left\|\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}\right\|$
5	Uchburchakning yuzi	$S_{uchburchak} = \frac{1}{2} \cdot \left\|\left[\vec{a} \cdot \vec{b}\right]\right\| = \frac{1}{2} \cdot \left\|\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}\right\|$
6	Parallelepipedning hajmi	$V_{para-ped} = \left\|\overline{a}\overline{b}\overline{c}\right\| = \left\|\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}\right\|$
7	Tetraedrning hajmi	$V_{tetraedr} = \frac{1}{6} \cdot \left\|\overline{a}\overline{b}\overline{c}\right\| = \frac{1}{6} \cdot \left\|\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}\right\|$

Glossariy

MATRITSA, ANIQLOVCHI VA CHIZIQLI TENGAMALAR SISTEMALARI
MATRITSA RANGINI ANIQLASH

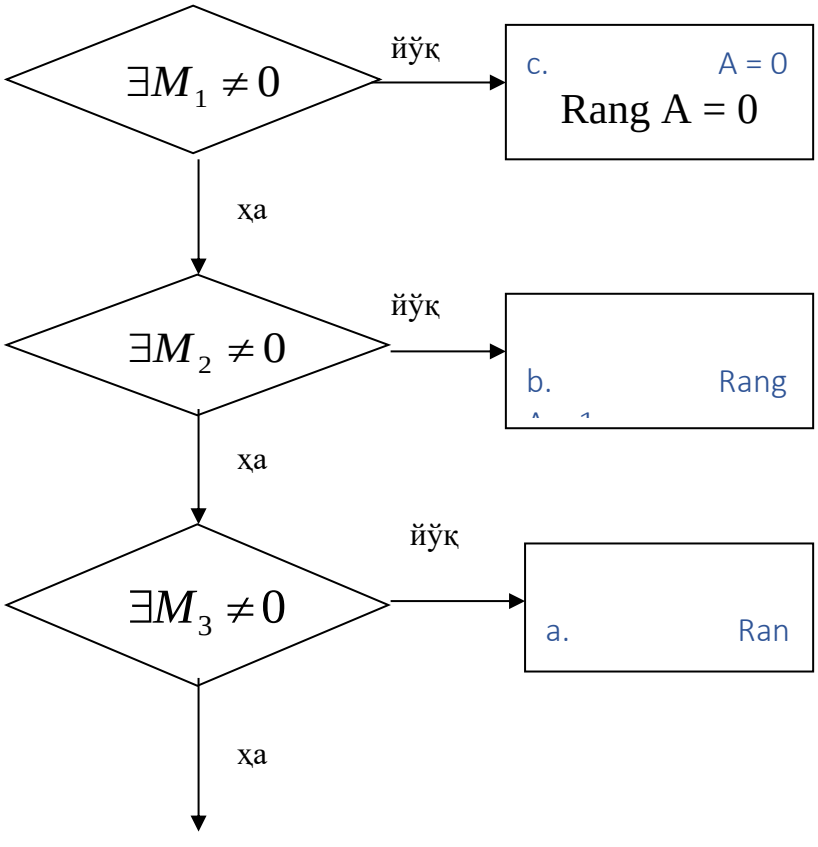
$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

MINORNING TA'RIFI

A matritsaning k –tartibli **minori M_k** deb uning 0 dan farqli ixtiëriy *k*-tartibli aniqlovchisiga aytiladi

$$M_1 = a_{ij}, \quad M_2 = \begin{vmatrix} a_{ij} & a_{ik} \\ a_{sj} & a_{sk} \end{vmatrix}, \quad u\ m.\partial.$$

$$i, s = 1,\dots,m, \quad j,k = 1,\dots,n$$



Matritsa rangining ta'rifi.

A matritsaning *r rangi deb, uning 0 dan farqli eng katta minori tartibi r* ga aytiladi:

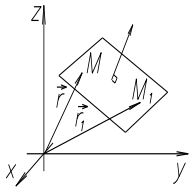
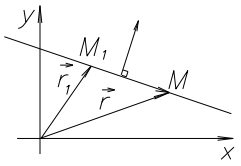
$\exists M_r \neq 0,\ \forall M_k = 0$ *ёки* $\overline{\exists} M_k, \quad k = r+1, r+2, \dots$

(0 dan farqli *r* tartibli minor mavjud, boshqa yuqori tartibli minorlar mavjud emas ёки qiymati nolga teng).

TO'G'RI CHIZIQ VA TEKISLIK GEOMETRIYASI JADVALLARDA

Uch o'lchovli R_3 fazoda R tekislik va ikki o'lchovli R_2 fazoda L to'g'ri chiziq tenglamalari

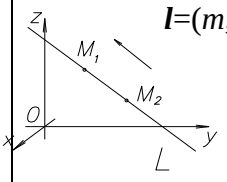
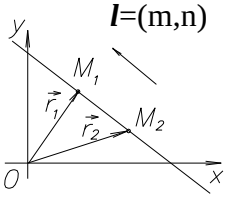
1-jadval

Uch o'lchovli R_3 fazoda R tekislikning koordinata shaklidagi tenglamasi	*P, L larning R_3 va R_2 dagi v*ektor ko'rinishdagi tenglamalari	*Ikki o'lchovli R_2 fazoda L to'g'ri chiziqning koordinata shaklidagi tenglamasi*
I R_3	***P* va *L larning berilgan* M_1 nuqtadan o'tib berilgan *N* *vektorga* perpendikulyar bo'lgan tenglamalari**	R_2 I
$\boldsymbol{N}=(A,B,C)$  $\boldsymbol{r} = (x,y,z)$ $\boldsymbol{r_1} = (x_1,y_1,z_1)$ $M_1(x_1,y_1,z_1)\in P$ $\forall M(x,y,z)\in P$ $\underline{A(x-x_1)+B(y-y_1)+C(z-z_1)=0}$	$\boldsymbol{r-r_1 = M_1M}$ $\boldsymbol{M_1M} \perp \boldsymbol{N}(P)$ $\boldsymbol{M_1M} \perp \boldsymbol{N}(L)$ $\underline{(\boldsymbol{r-r_1,N}) = 0}$ $\underline{(\boldsymbol{M_1M,N}) = 0}$ Vektorlarning perpendikulyarlik sharti	$\boldsymbol{N} = (A,B)$ $\boldsymbol{r} = (x,y)$ $\boldsymbol{r_1} = (x_1,y_1)$ $M(x_1,y_1) \in L$ $\forall M(x,y) \in L$  $\underline{A(x-x_1)+B(y-y_1) = 0}$
II R_3	**Umumiy tenglamalar**	R_2 II
$\underline{Ax + By + Cz + D = 0}$ $D = -Ax_1 - By_1 - Cz_1$	$\underline{(\boldsymbol{r},\boldsymbol{N}) + D = 0}$ $D = -(\boldsymbol{r_1}, \boldsymbol{N})$	$\underline{Ax + By + D = 0}$ $D = -Ax_1 - By_1$
III R_3	***n ta* fiksirlangan *M ta nuqta orqali***	R_2 III
n = 3 $M_1(x_1,y_1,z_1)\in P$, $\boldsymbol{M_1M}\in R$ $M_2(x_2,y_2,z_2)\in P$, $\boldsymbol{M_2M_1}\in R$ $M_3(x_3,y_3,z_3)\in P$, $\boldsymbol{M_3M_1}\in R$ $\forall M(x,y,z)\in P$ $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$	n = 3 $M_1 \in P, L$ $M_2 \in P, L$ $\forall M \in P, L$ $M_3 \in P$	n = 2 $M_1(x_1,y_1) \in L$, $M_2(x_2,y_2) \in L$ $\forall M(x,y) \in L$ $\boldsymbol{M_1M} \,\|\|\, \boldsymbol{M_2M_1}$ $\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x-x_1 & y-y_1 & 0 \\ x_2-x_1 & y_2-y_1 & 0 \end{vmatrix} = \bar{0}$
$A = \begin{vmatrix} y_2-y_1 & z_2-z_1 \\ y_3-y_1 & z_3-z_1 \end{vmatrix}$ $B = -\begin{vmatrix} x_2-x_1 & z_2-z_1 \\ x_3-x_1 & z_3-z_1 \end{vmatrix}$ $C = \begin{vmatrix} x_2-x_1 & y_2-y_1 \\ x_3-x_1 & y_3-y_1 \end{vmatrix}$ (1. I.)	$(\boldsymbol{M_1M},\boldsymbol{M_1M_2},\boldsymbol{M_1M_3})=0$ Vektorlarning komplanarlik sharti	$[\boldsymbol{M_1M},\boldsymbol{M_1M_2}] = \boldsymbol{0}$ Vektorlarning kollinearlik sharti $A = y_2-y_1$; $B = -(x_2-x_1)$, $\Leftrightarrow$ $A(x-x_1) + B(y-y_1) = 0$ (1. I.)
IV R_3	**Kesmalardagi tenglamalar**	R_2 IV
$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ $y=0, z=0 \Rightarrow x=a$	$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ $\boldsymbol{\tau} = \boldsymbol{i}/a + \boldsymbol{j}/b + \boldsymbol{k}/c$ $(\boldsymbol{r},\boldsymbol{\tau}) = 1$	$\frac{x}{a}+\frac{y}{b}=1$ $y=0 \Rightarrow x=a$

$x=0,\ z=0 \Rightarrow y=b$ $x=0,\ y=0 \Rightarrow z=c$	$\boldsymbol{\tau} = (1/a,\ 1/b,\ 1/c)$ $\lvert \boldsymbol{r} \rvert \cos(\boldsymbol{r},\boldsymbol{\tau}) = 1/\lvert \boldsymbol{\tau} \rvert$	$x=0 \Rightarrow y=b$

L to'g'ri chiziqning uch o'lchovli R_3 fazoda va ikki o'lchovli R_2 fazodagi tenglamalari

JADVAL 2

L to'g'ri chiziqningi R_3 dagi koordinata ko'rinishdagi tenglamasi	*L* to'g'ri chiziqningv R_2 va R_3 lardagi vektor ko'rinishdagi tenglamasi	*L to'g'ri chiziqningi* R_2 dagi koordinata ko'rinishdagi tenglamasi
I **L to'g'ri chiziqning kanonik tenglamasi** I		
$\boldsymbol{l}=(m,n,p)$  $\mathbf{l}=(m,n,p)\ \|\|\ L$ $M_1(x_1,y_1,z_1)\in L$ $M_2(x_2,y_2,z_2)\in L$ $\forall M(x,y,z)\in L$ $\frac{x-x_1}{m}=\frac{y-y_1}{n}=\frac{z-z_1}{p}$	$\boldsymbol{r}-\boldsymbol{r}_1=\boldsymbol{M_1M}\ \|\|\ \boldsymbol{l}$ $\boldsymbol{r}_2-\boldsymbol{r}_1=\boldsymbol{M_1M_2}\ \|\|\ \boldsymbol{l}$ $[\boldsymbol{r}-\boldsymbol{r}_1,\ \boldsymbol{l}\,]=0$ $[\boldsymbol{M_1M},\ \boldsymbol{l}\,]=0$	$\boldsymbol{l}=(m,n)$  $\boldsymbol{l}=(m,n)\ \|\|\ L$ $M_1(x_1,y_1)\in L$ $M_2(x_2,y_2)\in L$ $\forall M(x,y)\in L$ $\frac{x-x_1}{m}=\frac{y-y_1}{n}$
II **L to'g'ri chiziqning parametrik tenglamalari** II		
$\frac{x-x_1}{m}=\frac{y-y_1}{n}=\frac{z-z_1}{p}=t,\ \forall t\in R_1$ $\begin{cases} x=x_1+mt, \\ y=y_1+nt, \\ z=z_1+pt \end{cases}$	$\boldsymbol{r}-\boldsymbol{r}_1\ \|\|\ \boldsymbol{l},\ \forall t\in R_1$ $\boldsymbol{M_1M}\ \|\|\ \boldsymbol{l}$ $\boldsymbol{r}-\boldsymbol{r}_1=\boldsymbol{M_1M}=t\boldsymbol{l}$ $\boldsymbol{r}=\boldsymbol{r}_1+t\boldsymbol{l}$ $[\boldsymbol{M_1M},\ t\boldsymbol{l}\,]=0$	$\frac{x-x_1}{m}=\frac{y-y_1}{n}=t,\ \ \forall t\in R_1$ $\begin{cases} x=x_1+mt, \\ y=y_1+nt. \end{cases}$
III **L to'g'ri chiziqning ikkita M_1 va M_2 nuqtalar orqali o'tuvchi tenglamalari** III		
$\boldsymbol{l}\ \|\|\ L,\ \mathbf{l}=(m,n,p),\ \boldsymbol{l}t=\boldsymbol{M_1M_2}$ $m=x_2-x_1,\ n=y_2-y_1,\ p=z_2-z_1$ $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$	$\boldsymbol{M_1M}\ \|\|\ \boldsymbol{M_1M_2}\ \|\|\ \boldsymbol{l}$ $M_1\in L,\ M_2\in L,\ \forall M\in L$ $[\boldsymbol{M_1M},\boldsymbol{M_1M_2}]=0$	$\boldsymbol{l}\ \|\|\ L,\ \boldsymbol{l}=(m,n),\ t\boldsymbol{l}=\boldsymbol{M_1M_2}$ $m=x_2-x_1,\ n=y_2-y_1$ $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$

IV **L to'g'ri chiziqning R_3 da umumiy tenglamasi** $(P_1\cap P_2)$	V **L to'g'ri chiziqning R_2 dagi *k* burchak koeffi sientli tenglamasi**
$\boldsymbol{N_1}=(A_1,B_1,C_1)$ $\boldsymbol{N_2}=(A_2,B_2,C_2)\quad \boldsymbol{N_1}\ \|\|\ \boldsymbol{N_2}$ $L=\{P_1\cap P_2\}$ $\begin{cases} \boldsymbol{A_1x+B_1y+C_1z+D_1=0}, \\ \boldsymbol{A_2x+B_2y+C_2z+D_2=0}. \end{cases}$	$Ax+By+D=0,\ B\neq 0$ $\Downarrow$ $y=-\frac{A}{B}x-\frac{D}{B}$

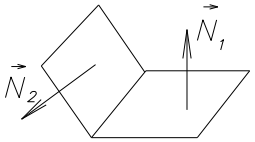
$N_1 \parallel N_2 \Leftrightarrow P_1 \cap P_2 \Leftrightarrow$ $Rang\begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{bmatrix} = 2.$	$\underline{y=kx+b}$ $\quad k = y' = -\frac{A}{B} = tg\alpha\ ,\ \alpha = \left(\bar{l}, \bar{i}\right)$ $\quad b = -\frac{D}{B}$ $\quad \alpha \geq 0$

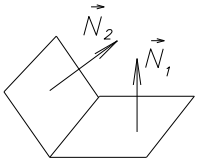
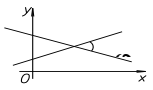
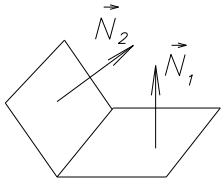
2*a Jadval* (2 jadvalning davomi)

***L* to'g'ri chiziqning R_3 dagi tenglamalari orasidagi bog'liqlik**	***L* to'g'ri chiziqning R_2 dagi tenglamalari orasidagi bog'liqlik**
Umumiy (2.IV) $\begin{cases} A_1x + B_1y + C_1z + D_1 = 0, \\ A_2x + B_2y + C_2z + D_2 = 0 \end{cases}$ $\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix} \neq 0 \Rightarrow z_0=0$ $\begin{cases} A_1x + B_1y + D_1 = 0, \\ A_2x + B_2y + D_2 = 0 \end{cases} \Rightarrow M_0(x_0,y_0,0) \in L$ ёки $\begin{vmatrix} A_1 & C_1 \\ A_2 & C_2 \end{vmatrix} \neq 0 \Rightarrow y_1=0 \Rightarrow M_1(x_1,0,z_1) \in L$ ёки $\begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix} \neq 0 \Rightarrow x_2=0 \Rightarrow M_2(0,y_2,z_2) \in L$ $N_1=(A_1,B_1,C_1),$ $N_2=(A_2,B_2,C_2)$ $\Leftrightarrow$ $l=[N_1,N_2]=(m,n,p)$ $t = \frac{x-x_0}{m} = \frac{y-y_0}{n} = \frac{0-z_0}{p}$ – **kanonik** (2.I) $\begin{cases} x=x_0+mt, \\ y=y_0+nt, \\ z=0+pt \end{cases}$ – **parametrik** (2.II) $\Downarrow$ $\frac{x-x_0}{x_1-x_0} = \frac{y-y_0}{y_1-y_0} = \frac{0-z_0}{z_1-z_0}$ – $M_0 \in L, M_1 \in L$ **ikki nuqta orqali** (2.III) $\begin{cases} (x-x_0)(y_1-y_0) = (y-y_0)(x_1-x_0), \\ (y-y_0)(z_1-z_0) = (0-z_0)(y_1-y_0) \end{cases}$ – **umumiy** (2.IV)	**Burchak koeffitsientli**: (2.V) $y=kx+b$ $\quad \vec{N} = (k,-1),$ $\Downarrow$ $\begin{cases} x=x_1,\ y=kx_1+b=y_1 \Rightarrow M_1(x_1,y_1) \in L, \\ x=x_2,\ y=kx_2+b=y_2 \Rightarrow M_2(x_2,y_2) \in L. \end{cases}$ $k = \frac{y_2-y_1}{x_2-x_1}$ $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}$ **$M_1 \in L$ i $M_2 \in L$ nuqtalar orqali o'tuvchi** (1.III) (2.III) $\begin{cases} x_2-x_1=m, \\ y_2-y_1=n \end{cases} \Rightarrow \frac{x-x_1}{m} = \frac{y-y_1}{n}$ – **kanonik** (2.I) $\Downarrow$ $\begin{cases} n(x-x_1)=m(y-y_1) \\ n(x-x_1)-m(y-y_1)=0, \\ n=A;\ -m=B; \end{cases}$ $A(x-x_1)+B(y-y_1)=0$ $\quad M_1(x_1,y_1) \in L$ $N=(A,B) \perp L$ (1.I) $\Downarrow$ $-Ax_1-By_1=D$ $Ax+By+D=0$ – **umumiy** (1.II)

P TEKISLIKLARNING UCH O'LCHOVLI R_3 FAZODA VA *L* TO'G'RI CHIZIQLARNING IKKI O'LCHOVLI R_2 FAZODA O'ZARO JOYLASHISHI

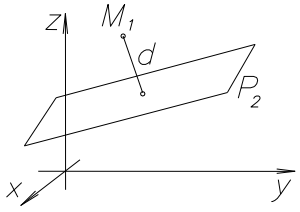
3-jadval

I 2-jadvalda qabul qilgan belgilashlar {P1,P2} R_3	I	2-jadvalda qabul qilgan I belgilashlar {L1,L2} R_2
$P1: A_1x + B_1y + C_1z + D_1 =$ $P2: A_2x + B_2y + C_2z + D_2 =$ $N_1=(A_1,B_1,C_1);$ $N_2=(A_2,B_2,C_2)$	R_3 $\quad cos\ \varphi = \frac{(\vec{N}_1,\vec{N}_2)}{\lvert\vec{N}_1\rvert\lvert\vec{N}_2\rvert}$ 	a) $\left.\begin{matrix} L1: A_1x + B_1y + D_1 = 0, \\ L2: A_2x + B_2y + D_2 = 0 \end{matrix}\right\} \chi$ $N_1=(A_1,B_1);$ $N_2=(A_2,B_2)$ $\text{Rang}\begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix} = \text{Rang A}(\chi)$

Rang $\begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{bmatrix}$ $= \text{Rang A}(\psi)$ Rang $\begin{bmatrix} A_1 & B_1 & C_1 & D_1 \\ A_2 & B_2 & C_2 & D_2 \end{bmatrix}$ $= \text{Rang B}(\psi)$ 	$N_1=(A_1, B_1, C_1)$; $N_2=(A_2, B_2, C_2)$ R_2  $N_1=(A_1,B_1)$ $N_2=(A_2,B_2)$	Rang $\begin{bmatrix} A_1 & B_1 & D_1 \\ A_2 & B_2 & D_2 \end{bmatrix}$ $= \text{Rang B}(\chi)$ b)$L1$: $y=k_1x+b_1$ $L2$: $y=k_2x+b_2$ $tg\varphi = \left\lvert\frac{k_2-k_1}{1+k_1k_2}\right\rvert$ $\Downarrow$ $k_1=tg\ \alpha_1$; $k_2=tg\ \alpha_2$ $tg\varphi = tg\ (\alpha_2-\alpha_1) = \frac{tg\alpha_2 - tg\alpha_1}{1+tg\alpha_1 tg\alpha_2}$,
II	**{P1, P2} tekisliklar va {L1, L2} to'g'ri chiziqlarning o'zaro joylashishi**	
	I	
R_n;n=3 da $\{R1, R2\}$ tekisliklar	*P va L lar qanday joylashgan*	R_n; n=2 da $\{L1, L2\}$to'g'ri chiziqlar
$P1 \cap P2$ (**kesishadi**) $cos\varphi=\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\neq\pm 1$ $P1\perp P2 \Leftrightarrow \mathbf{N_1}\perp\mathbf{N_2} \Leftrightarrow cos\varphi=0$ $\{P1\cap P2\}=L,\ L\in P1,\ L\in P2$ *Rang A*(ψ)= = *Rang B*(ψ)= 2 < 3=*n* birgalikda aniqlanmagan sistema (ψ)	$\underbrace{P1 \cap P2, L1 \cap L2}$ $\mathbf{N_1}\|\|\not{\mathbf{N_2}}$ $\varphi \neq \pi k$, k=0, ±1, ±2, ... $cos\ \varphi \neq +1$ 	$L1\cap L2$ (**kesishadi**) a)$cos\ \ \phi = \frac{A_1A_2+B_1B_2}{\sqrt{A_1^2+B_1^2}\sqrt{A_2^2+B_2^2}} \neq \pm 1$ $L1\perp L2 \Leftrightarrow \mathbf{N}_1\perp\mathbf{N}_2 \Leftrightarrow cos\varphi = 0$ b) $tg\varphi=\frac{k_2-k_1}{1+k_1k_2}\neq 0$ $1+ k_1k_2 \neq 0$ $L1\perp L2 \Leftrightarrow 1+ k_1k_2 =0 \Leftrightarrow$ $\Leftrightarrow k_2= -1/k_1$ $\{L1\cap L2\}=M,\ M\in L1,\ M\in L2$ *Rang A*(χ)= = *Rang B*(χ)= 2=*n* birgalikda aniqlangan sistema (χ)
$P1\|\|P2$ (**parallel**) $\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}\neq\frac{D_1}{D_2}$ $cos\varphi = \pm 1$	$\underbrace{P1\|\|P2, L1\|\|L2}$ $\mathbf{N_1}=\lambda\mathbf{N_2}$; $D_1\neq\lambda D_2$ $\lambda\in R_1$ 1=*Rang A*(ψ,χ) < < *Rang B*(ψ,χ) = 2 (ψ),(χ)sistemalar birgalikda emas	$L1\|\|L2$ (**parallel**) *a*)$\frac{A_1}{A_2}=\frac{B_1}{B_2}\neq\frac{D_1}{D_2}$ $cos\varphi = \pm 1$ *b*) $k_1 = k_2$; $b_1 \neq b_2$ $tg\varphi = 0$
$P1\equiv P2$ (**ustma-ust tushadi**) $\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=\frac{D_1}{D_2}=\lambda \in R_1$ $cos\varphi = \pm 1$	$\underbrace{P1 \equiv P2, L1 \equiv L2}$ $\mathbf{N_1}=\lambda\mathbf{N_2}$; $D_1 = \lambda D_2$; $\lambda\in R_1$ *Rang A*(ψ,χ)= = *Rang B*(ψ,χ)= 1 birgalikda aniqlanmagan sistemalar (ψ), (χ)	$L1\equiv L2$ (**ustma-ust tushadi**) *a*)$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{D_1}{D_2}=\lambda\in R_1$ $cos\varphi = \pm 1$ *b*) $k_1 = k_2$, $b_1 = b_2$ $tg\varphi = 0$

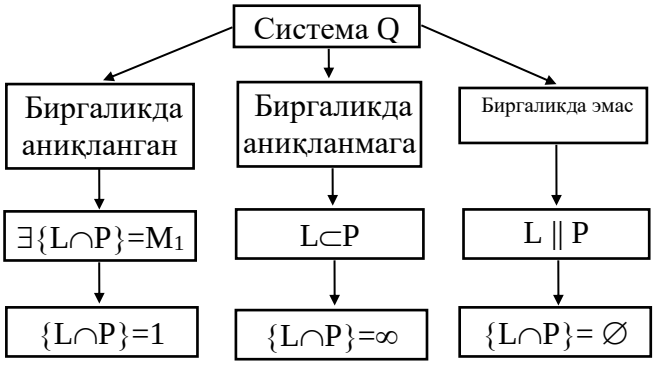
R_3 da $P1$ va $P2$ tekisliklar orasidagi masofa $d(P1,P2)$ va $L1$ va $L2$ to'g'ri chiziqlar orasidagi masofa $d(L1,L2)$, R_3 da P *tekislik va to'g'ri chiziq* L *ning kesishishi* $\{P\cap L\}$

4 jadval

I **$P1 \parallel P2, L1 \parallel L2$ v R_3 koordinata shaklda**	**$P1 \parallel P2, L1 \parallel L2$, $\bar{N}_1 \| \bar{N}_2, \bar{l}_1 \| \bar{l}_2$ vektor shaklda**	**$L1 \parallel L2$ v R_2 koordinata shaklda** II
P1: $Ax+By+Cz+D_1=0$ P2: $\underbrace{Ax+By+Cz+D_2=0}$, $D_1 \neq D_2$ $\bar{N}_1=\bar{N}_2=\bar{N}=(A,B,C)\perp P1,P2$ $\Leftarrow$ $d(P1,P2)=d(M_1,P2)=d(M_2,P1)$ $=$ $=\frac{\lvert A(x_2-x_1)+B(y_2-y_1)+C(z_2-z_1)\rvert}{\sqrt{A^2+B^2+C^2}}$ $M_1(x_1,y_1,z_1)\in P1,\ M_2(x_2,y_2,z_2)\in P2$	$\bar{N}=(A,B,C)\quad \bar{l}=(m,n,p)$ $d(P1,P2)=d(M_1,P2)=$ $d(M_2,P1)=d(L1,L2)=$ $=d(M_1,L2)=d(M_2,L1)=$ $=\left\lvert np_{\bar{N}}\overrightarrow{M_1M_2}\right\rvert$ $=\frac{\left\lvert(\overrightarrow{M_1M_2},\bar{N})\right\rvert}{\lvert\bar{N}\rvert}$  	L1: $Ax+By+D_1=0$ L2: $\underbrace{Ax+By+D_2=0}$ $\bar{N}_1=\bar{N}_2=\bar{N}=(A,B)\perp L1,L2$ $\Rightarrow$ $d(L1,L2)=d(M_1,L2)=d(M_2,L1)=$ $=\frac{\lvert A(x_2-x_1)+B(y_2-y_1)\rvert}{\sqrt{A^2+B^2}}$ $M_1(x_1,y_1)\in L1,\ M_2(x_2,y_2)\in L2$
$L1:\frac{x-x_1}{m}=\frac{y-y_1}{n}=\frac{z-z_1}{p}$ $L2:\frac{x-x_2}{m}=\frac{y-y_2}{n}=\frac{z-z_2}{p}$ $\bar{l}_1=\bar{l}_2=\bar{l}=(m,n,p)$ $M_1(x_1,y_1,z_1)\in L1$ $M_2(x_2,y_2,x_2)\in L2$ $d(L1,L2)=d(M_1,L2)=d(M_2,L1)=$ $=\frac{mod\begin{vmatrix}\bar{i}&\bar{j}&\bar{k}\\x_2-x_1&y_2-y_1&z_2-z_1\\m&n&p\end{vmatrix}}{\sqrt{m^2+n^2+p^2}}\Leftarrow$	$d(L1,L2)=d(M_1,L2)$ $=d(M_2,L1)$ $=$ $=h_{\Delta\overrightarrow{M_1M_{22}},\bar{l}}=\frac{\left\lvert\left[\overrightarrow{M_1M_2},\bar{l}\right]\right\rvert}{\lvert\bar{l}\rvert}h-$ uchburchakning balandligi	$L1:\frac{x-x_1}{m}=\frac{y-y_1}{n}$ $L2:\frac{x-x_2}{m}=\frac{y-y_2}{n}$ $\bar{l}_1=\bar{l}_2=\bar{l}=(m,n,0)$ $M_1(x_1,y_1)\in L1$ $M_2(x_2,y_2)\in L2$ $\Rightarrow d(L1,L2)=d(M_1,L2)=$ $d(M_2,L1)=$ $=\frac{mod\begin{vmatrix}\bar{i}&\bar{j}&\bar{k}\\x_2-x_1&y_2-y_1&0\\m&n&0\end{vmatrix}}{\sqrt{m^2+n^2}}$

III $L1$ va $L2$ to'g'ri chiziqlar R_3 da chatishadi $P1 \parallel P2$ $(L1\subset P1, L2\subset P2)$	L to'g'ri chiziq va P *tekislik* R_3 da kesishadi IV $\{P\cap L\}=M_1$
$L1:\underbrace{\frac{x-x_1}{m_1}=\frac{y-y_1}{n_1}=\frac{z-z_1}{p_1}}_{L1\|\bar{l}_1=(m_1,n_1,p_1),\ M_1(x_1,y_1,z_1)\in L1}$ $L2:\underbrace{\frac{x-x_2}{m_2}=\frac{y-y_2}{n_2}=\frac{z-z_2}{p_2}}$ $L2\parallel\bar{l}_2=(m_2,n_2,p_2),M_2(x_2,y_2,z_2)\in L2$ $d(L1,L2)=d(M_1,L2)=d(M_2,L1)=h_{\Pi(\bar{M}_1\bar{M}_2,\bar{l}_1\bar{l}_2)}=$ $=\frac{V_{\Pi(\bar{M}_1\bar{M}_2,\bar{l}_1\bar{l}_2)}}{S_{\Delta\ \bar{l}_1,\bar{l}_2}}=\frac{\left\lvert(M_1M_2,\bar{l}_1,\bar{l}_2)\right\rvert}{\left\lvert\left[\bar{l}_1,\bar{l}_2\right]\right\rvert}=$ $=\frac{\mathrm{mod}\begin{vmatrix}x_2-x_1&y_2-y_1&z_2-z_1\\m_1&n_1&p_1\\m_2&n_2&p_2\end{vmatrix}}{\mathrm{mod}\begin{vmatrix}\bar{i}&\bar{j}&\bar{k}\\m_1&n_1&p_1\\m_2&n_2&p_2\end{vmatrix}}\neq 0$ (d($L1,L2$)=0$\Leftrightarrow L1\cap L2$);P($\boldsymbol{M_1M_2,l_1,l_2}$) – $\boldsymbol{M_1M_2}$, $\boldsymbol{l_1,l_2}$, - vektorlarga qurilgan parallelepiped, h – uning balandligi	P: $\underbrace{Ax+Bx+Cz+D=0}_{P\perp\bar{N}=(A,B,C)\perp P}$ L: $\underbrace{\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}}_{L\|\bar{l}=(m,n,p)}$ $M_0(x_0,y_0,z_0)\in L$ $\cos(\bar{N},\bar{l})=\cos(\frac{\pi}{2}-\varphi)=\sin\varphi$ $\sin\varphi=\cos(\bar{N},\bar{l})=\frac{(\bar{N},\bar{l})}{\lvert\bar{N}\rVert\bar{l}\rvert}=$ $=\frac{Am+Bn+Cp}{\sqrt{A^2+B^2+C^2}\sqrt{m^2+n^2+p^2}}\neq\pm1$ $\left.\begin{array}{l}sin\,\phi=\pm1\\cos\,\phi=0\\(\bar{N},\bar{l})=0\end{array}\right\}\ (P\|L)\cup(L\subset P)$ $sin\ \varphi=0\Leftrightarrow L\perp P,\ \boldsymbol{l}\,\|\,\boldsymbol{N}$

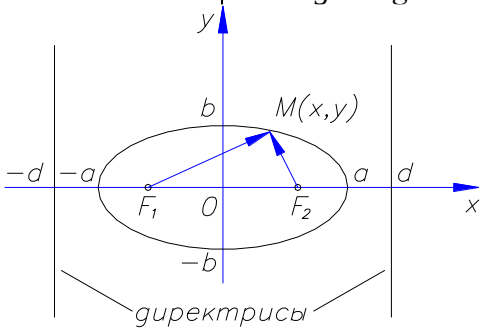
4a-jadval (4-jadvalning davomi)

VI P tekislik va L *to'g'ri chiziqning* R_3 da: $(P\perp L)$ ortogonallik, $(P\|\|L)$ kollinearlik shartlarining hamda P va L $(P\cap L)$ kesishish shartining vektor shakli.	$\{P\cap L\}=M_1(x_1,y_1,z_1)$ – *P tekislik va* L to'g'ri chiziqning V R_3 kesishishi
1. $L\perp P \Leftrightarrow [\bar{l},\bar{N}]=\vec{0}$ 2. $L\|\|P \Leftrightarrow (\bar{l},\bar{N})=0$ 3. $L\cap P$ $\Leftrightarrow \left.\begin{matrix} L:\ \bar{r}=r_0+t\bar{l} \quad (2.II) \\ P:\ (\bar{r},\bar{N})+D=0 \quad (1.II) \end{matrix}\right\} \Rightarrow$ $\Rightarrow (\bar{r}_0+t\bar{l},\bar{N})+D=0,$ $t=-\frac{(\bar{r}_0,\bar{N})+D}{(\bar{l},\bar{N})}=t_1 \Leftrightarrow M_1(t_1)=\{L\cap P\}$	$\left.\begin{matrix} P:\ Ax+By+Cz+D=0 \\ L:\ \frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}=t \end{matrix}\right\}\ Q;$ $P\perp\bar{N}=(A,B,C)$ $M_0(x_0,y_0,z_0)\in LA(x_0+$ $L\|\|\bar{l}=(m,n,p)$ $mt)+B(y_0+nt)+C(z_0+pt)+D=0 \Rightarrow t=t_1$ (2.II) $\Rightarrow$ $\begin{cases} x_1=x_0+t_1m \\ y_1=y_0+t_1n \\ z_1=z_0+t_1p \end{cases} \Rightarrow \begin{matrix} M_1(x_1,y_1,z_1)\in L, \\ M_1(x_1,y_1,z_1)\in P \end{matrix}$ 

IKKINCHI TARTIBLI CHIZIQLAR

Ellipsning ta'rifi

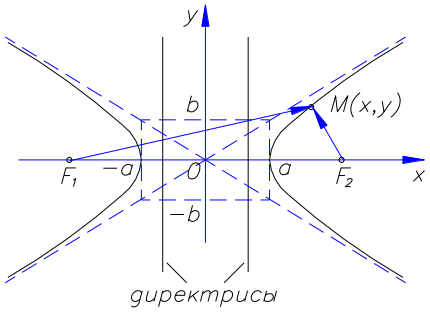
Ellips deb tekislikning shunday nuqtalar to'plamiga aytiladiki, bu nuqtalardan shu tekislikda ётuvchi ellipsning fokuslari deb ataluvchi ikkita tayin nuqtalargacha bo'lgan **masofalar yig'indisi** o'zgarmas son, fokuslar orasidagi masofadan katta va ***2a*** *ga teng bo'lsa*.



a – ellipsning katta yarim o'qi; b – ellipsning kichik yarim o'qi; $F_1(-s,0)$ *va* $F_2(s,0)$ – ellipsning fokuslari; $c^2=a^2-b^2$, s – ellipsning fokus masofasi; $\varepsilon=\frac{c}{a}<1$, ε – ellipsning ekssentrisiteti; $\vec{r_1}=\vec{F_1M}$, $\vec{r_2}=\vec{F_2M}$ – fokal radius-vektorlar; ta'rifga ko'ra $r_1+r_2=2a$. $x=\pm\frac{a}{\varepsilon}=\pm\ d$ to'g'ri chiziqlar ellipsning direktrisalari deyiladi.

Ellipsning **kanonik** tenglamasi $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

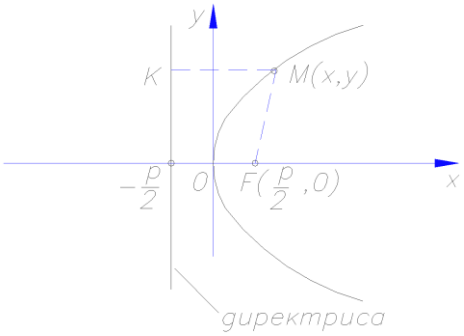
Giperbolning ta'rifi	**Giperbola** deb tekislikning shunday nuqtalar to'plamiga aytiladiki, ular uchun fokuslar deb ataluvchi ikkita tayn nuqtalargacha bo'lgan **masofalar ayirmasi moduli *modul*** *tayin son, fokuslar orasidagi masofadan kichik va* ***2a*** *ga teng bo'lsa.*



a – giperbolaning haqiqiy yarim o'qi; b – giperbolaning mavhum yarim o'qi; $F_1(-s,0)$ i $F_2(s,0)$ – giperbolaning fokuslari; $c^2=a^2+b^2$, s – giperbolaning fokuslari orasidagi masofa; $\varepsilon=\frac{c}{a}>1$, $\varepsilon-$ giperbolaning ekssentrisiteti; $\vec{r_1}=\vec{F_1M}$, $\vec{r_2}=\vec{F_2M}$ – fokal radius-vektorlar; tap'rifga ko'ra $|r_1-r_2|=2a$. $x=\pm\frac{a}{\varepsilon}=\pm d$ to'g'ri chiziqlar giperbolaning direktrisalari, $y=\pm\frac{b}{a}x$ giperbolaning asimptotlari tenglamalari .

Giperbolaning **kanonik** tenglamalari $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

Parabolaning ta'rifi	**Parabola** deb tekislikning shunday nuqtalar to'plamiga aytiladiki, ularning har biri fokus deb ataluvchi tayin bitta **nuqtadan** va fokusdan o'tmaydigan direktrisa deb ataluvchi berilgan **to'g'ri chiziqdan bir xil masofada** joylashgan bo'lsa.



Parabolaning **kanonik** tenglamasi: $y^2=2px$.

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